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# THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY  
AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE

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JULY 1924

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NUMBER 1

## THE SPECTRUM OF HELIUM IN THE EXTREME ULTRA-VIOLET

By THEODORE LYMAN

### ABSTRACT

*Extreme ultra-violet spectra of helium to 256 Å.*—The introduction deals with the effect of the nature of the ruling of the diffraction grating on the intensity of spectra in the extreme ultra-violet and on the strength of ghosts. A method is given for producing a continuous spectrum extending to 900 Å.

The existence of the second enhanced series has been confirmed; and two members of the first enhanced series at 303.6 and 256.3 have been discovered.

Seven members of the *OS-mP* series have been measured. The identification of the line at  $\lambda 600.3$  corresponding to the second resonance potential is still in doubt. A line at  $\lambda 591.5$ , corresponding to a jump from an energy-level in the doublet system to the lowest level in the singlet system, has been discovered. The presence of a continuous spectrum extending from the limit of the *OS-mP* series is noted. The apparatus and methods are described; this section includes some remarks on oil-coated plates and on the probable advantages to be gained by spattering a speculum diffraction grating with silicon.

The investigation of the helium spectrum in the extreme ultra-violet has engaged my attention for several years, and some of the most interesting conclusions have already appeared in print,<sup>1</sup> but it is only recently that I have been able to place all the results on a basis whose solidity is satisfactory to me.

The investigation was carried on with concave diffraction gratings, and its results are influenced by the peculiarities of these instruments. It cannot be too strongly emphasized that in the examination of an unknown spectral region the reality of all the

<sup>1</sup> *Philosophical Magazine*, 41, 814, 1921; *Science*, 56, 167, 1922.

lines obtained with any one grating cannot be assumed; the ghosts or false spectra which occur in even the best instruments being a source of error which cannot be neglected. It is absolutely essential that the spectrum under consideration should be examined with several gratings ruled under different conditions and, if possible, on different engines. The difficulty of obtaining a sufficient number of such gratings has delayed the completion of the present research for a considerable period. In fact, had it not been for the activity and generosity of Professor R. W. Wood, the work could not have been completed even now.

Before proceeding to the more important results of the investigation it will be well to mention some of the by-products. In the first place, by comparing the performance of several gratings under exactly similar conditions, I have verified the statement of Millikan, Simeon, and others; for it appears that a lightly ruled grating gives a stronger spectrum in the region of extremely short wave-lengths than one ruled with a deeper groove.

In the second place, I have observed that two gratings, though presenting a considerable difference in depth of ruling, show false spectra or "Lyman ghosts" of identical distribution and nearly identical intensity. This result is important, for it confirms the statement of a preceding paragraph that in order to eliminate the errors due to these false spectra it is not only necessary to repeat observations with several different gratings, but that it is absolutely essential that the instruments should be ruled on different engines or at least under very different conditions. The matter has been illustrated by Wood, who by making an improvement in the driving mechanism of the dividing engine has succeeded in greatly reducing the intensity of the false spectra and in profoundly altering their distribution.

Thirdly, there is a discovery which though not directly related to the subject of this paper is yet of sufficient practical importance to merit special attention; it concerns a method of producing a continuous spectrum in the Schumann region. I find that if a very large capacity of the same order of magnitude as that used by Anderson in his work on the explosion of wires be discharged through a pyrex capillary tube, a continuous spectrum results, extending

from the less refrangible limit of my plates near  $\lambda 1900$  to the neighborhood of  $\lambda 900$ , and upon which certain lines appear as reversals. The lines of the helium spectrum hardly appear at all with this type of excitation.

#### ENHANCED SPECTRUM

We may now proceed to the chief results of the investigation. Some time ago I published a note<sup>1</sup> on the series in the enhanced spectrum of helium, whose lines follow the formula  $4N(1/2^2 - 1/m^2)$ , but I was not at the time entirely satisfied that the observed radiations had been rigorously proved to belong to the gas in question. The difficulty lay in the fact that the results were obtained with a disruptive discharge passing in a capillary tube, a mode of excitation which makes the exclusion of the lines due to impurities almost impossible. The series under examination has its first member at 1640.49, its second at 1215.18, and its third at 1084.98. The work of McLennan<sup>2</sup> on the subject does not seem to me conclusive. He was unable to observe the first line of the series at all, and the agreement of the other lines which he obtained may well have been fortuitous, for as he employed a disruptive discharge he was exposed to the confusing effect of impurities which I have just mentioned. In a note called forth by a statement of Hicks, I have pointed out<sup>3</sup> the grave danger of classifying lines as belonging to a given series unless due weight is given to the experimental circumstances under which the results were obtained.

I have endeavored to put the matter beyond question by employing a continuous current and a vacuum tube of a special design, due to Paschen.<sup>4</sup> In this type of tube the discharge from the interior of a hollow cylindrical cathode is viewed end on. Paschen has shown that an arrangement of this kind is particularly suitable for bringing out enhanced lines in helium, and I have found that those impurities which are torn from the walls with a disruptive discharge are absent and that the trace of hydrogen so often found in helium may be eliminated. With this apparatus I have observed the first member of the enhanced series at  $\lambda 1640.4$ , in sufficient strength.

<sup>1</sup> *Science*, **50**, 481, 1919.

<sup>2</sup> *Transactions of the Royal Society of Canada*, **15**, sec. 3, p. 15, 1921.

<sup>3</sup> *Nature*, **104**, 565, 1920.

<sup>4</sup> *Ann. d. Physik* (4), **71**, 142, 1923.

I feel therefore that the doubts which I entertained as to my previous results have been removed.

#### SERIES IN ARC SPECTRA

Some of the most important matters to which I wish to call the reader's attention are illustrated in Plate I, in which the upper spectrum covers the whole region between  $\lambda 480$  and  $\lambda 620$ , while the lower begins near  $\lambda 530$ . In the first place, the existence of the series whose first member lies at  $\lambda 584.4$  has been completely confirmed. It has been observed with at least four different gratings and under a variety of conditions of tube excitation. A continuous current has been usually employed for reasons already explained, and the lower spectrum in the illustration was obtained under these conditions; the upper spectrum, however, was taken with a disruptive discharge.

The line at  $584.40$  is of very great strength, and is accompanied by six new lines, the first three of which lie at  $537.12$ ,  $522.21$ , and  $515.65$ , whose intensities decrease with their wave-length and in a manner strongly suggesting a series relation. Luckily these first three members appear in the second-order spectrum of some gratings. A comparison with the hydrogen line  $1215.68$  and with the three following lines of the same series is therefore possible, with the result that the wave-lengths are probably correct to one or two tenths of a unit.

The spacing of these seven helium lines on the frequency scale is of great interest and importance, for it is found to be identical with the spacing of the first seven lines in the singlet principal series. It may be stated therefore with considerable certainty that the line  $584$  forms the first member of a principal series which, according to the notation of Professor Fowler, is to be represented by  $OS-mP$ .

On the less refrangible side of  $\lambda 584.4$  a prominent line lies at  $\lambda 600.3 \pm .6$ ; it is of a diffuse character, and under some circumstances there is a suggestion of a continuous prolongation extending toward the region of longer wave-lengths to the neighborhood of  $\lambda 618$ . It has been obtained with all five of the gratings used in the investigation, and, what is even more important, its intensity

PLATE I





relative to the members of the  $OS-mP$  series does not appear to change greatly as the purity of the helium is increased. However, the origin of this line is still not above suspicion, the doubt arising on the experimental side from the fact that it has been impossible to free the gas under consideration from the last trace of hydrogen and perhaps of other impurities. In fact, Millikan<sup>1</sup> gives a faint line in the spectrum of carbon at  $\lambda$  600.2 and a line in the spectrum of oxygen of intensity (5) at  $\lambda$  599.5; but the nature of our line is by no means established by this evidence since the strong oxygen lines at  $\lambda\lambda$  703.1, 718.5, and 834.0, and all the very strong lines of carbon, such as  $\lambda\lambda$  904.1, 977.1, and 1175.6, are entirely absent from my plates. There are also doubts based on theory, for if this line is due to helium it can be represented nearly, though not quite exactly, by the expression  $OS-1S$ , and thus should correspond to a jump between energy-levels, uncommon, and associated when it does occur in other elements with a violent mode of excitation; whereas  $\lambda$  600.3 is fairly strong and is obtained with a continuous current.

#### INTERSYSTEM COMBINATION

The next point, and one of very considerable interest, is concerned with the line at  $\lambda$  591.56; this radiation is clearly seen on the lower spectrum in Plate I. It may be represented by the relation  $OS-1\pi$  within the limits of error of measurement; it is interesting because it furnishes the first experimental evidence for a radiation from helium involving a so-called intersystem combination, that is to say, a jump from a doublet energy-level to the lowest singlet level. It appears that no one has found a resonance potential corresponding to this wave-length. This radiation is strongest with a continuous current; it has been found with all four of the brightest gratings. I think it unlikely that it is due to an impurity.

The illustration shows another phenomenon which may possibly prove of theoretical interest, namely, the continuous spectrum which follows the short wave-length limit of the  $OS-mP$  series and extends to the neighborhood of  $\lambda$  400. Now, curiously enough, this effect is only obtained with a disruptive discharge of a particular type, and the proof of its connection with helium and with the series

<sup>1</sup> *Physical Review*, **23**, 1, 1924.

which precedes it is beset by all the pitfalls which must ever accompany this kind of excitation.

#### FIRST ENHANCED SERIES

We began with the confirmation of the existence of the second enhanced series in helium; we may conclude the enumeration of the results of the investigation by the announcement of the discovery of two members of the first, and probably the most important, enhanced series  $4N(1-1/m^2)$ ; they lie, as they should, at  $\lambda$  303.6 and  $\lambda$  256.3. They have been obtained, to be sure, with a disruptive discharge, but the agreement of the measured wave-lengths with the predicted values and the fact that the lines occur in the spectra from three different gratings lead me to place great confidence in their reality.

Before turning to a detailed description of apparatus and methods, it may be well to remind the reader of the effect of some of these spectroscopic measurements on the work of a number of investigators in a different field. This connection arises in the following manner.

#### RADIATION AND IONIZATION POTENTIALS

The relation between the values of the resonance and ionization potentials in helium as formerly accepted and the wave-lengths of the  $OS-mP$  series was rather puzzling. The ionization potential should certainly correspond to the limit of the series; now this limit can be accurately calculated—it corresponds to 24.5 volts, but the experimental value was 25.3 volts. This was the chief difficulty, but it was not the only one, for the agreement between the wave-lengths of the individual spectrum lines and the values of the resonance potentials as determined by Franck and Knipping was not satisfactory. A correction of about  $-0.8$  volts if applied to all the potential measurements will bring the two sets of data into fair agreement, but at the expense of the first resonance potential at 19.7 volts, which is left without any corresponding line in the spectrum. Considerations of this nature have led Franck<sup>1</sup> to review the interpretation of his experimental data with the result that he has discovered a source of error; its removal involves a correction

<sup>1</sup> *Zeits. f. Phys.*, **11**, 155, 1922.

factor of the order just mentioned. When this factor has been applied, most of the values of the directly determined resonance and ionization potentials agree with the results of spectroscopic investigation. It is interesting to note, however, that Franck has found no potential corresponding to the line 591.5.

#### APPARATUS AND METHODS

This investigation has been carried on with two gratings of 50-cm radius and with three of 1 meter. The apparatus in which the instruments of short focus were used has been fully described,<sup>1</sup> but as my most recent spectroscope has proved to be a considerable improvement over previous patterns, it may be well to give some account of it, particularly in view of the increasing interest in vacuum spectroscopy. It will be well to emphasize the fact that the success of this type of instrument depends chiefly on the small dimensions of the slit and the arrangement of vacuum pumps whereby the body of the spectroscope is kept at a good vacuum while the pressure in the discharge tube may be several millimeters of mercury.

The end of the apparatus which contains the grating need not particularly concern us; it is closed by a vertical brass plate made air-tight by a ring of soft wax. Most of the adjustments for focus are made by removing this plate and manipulating the grating.

The arrangement of the slit and the plate-holder end of the instrument presents some novelties of design which are illustrated in the drawing of a horizontal section (Fig. 1).

The body of the spectroscope consists of a drawn brass tube of 14.9 cm internal diameter which carries at the grating end a flange and to which, at the other extremity, is permanently soldered the plate *A*. This plate carries the slit tube *B*, and from it protrudes the rectangular brass box *C*, 14 cm long, 7.8 cm high, and 5.5 cm wide. The brass plate *D* upon which the plate-holder is mounted slips into this box and rests upon its floor; a spring *G* pressing on the back of the box holds the base plate in position. The opening at the end of *C* is surrounded by a flange, and is closed by a plate *E*, the joint being made air-tight by a ring of soft wax. To facilitate

<sup>1</sup> *Journal of the American Optical Society*, 7, 495, 1923.

adjustment there is a slot in the back of *C* covered with a glass plate, *W*. This window is made light-tight by a cap.

The plate-holder slides in vertical ways on the back of the vertical plate *F*; this plate is so mounted that it may be turned about a

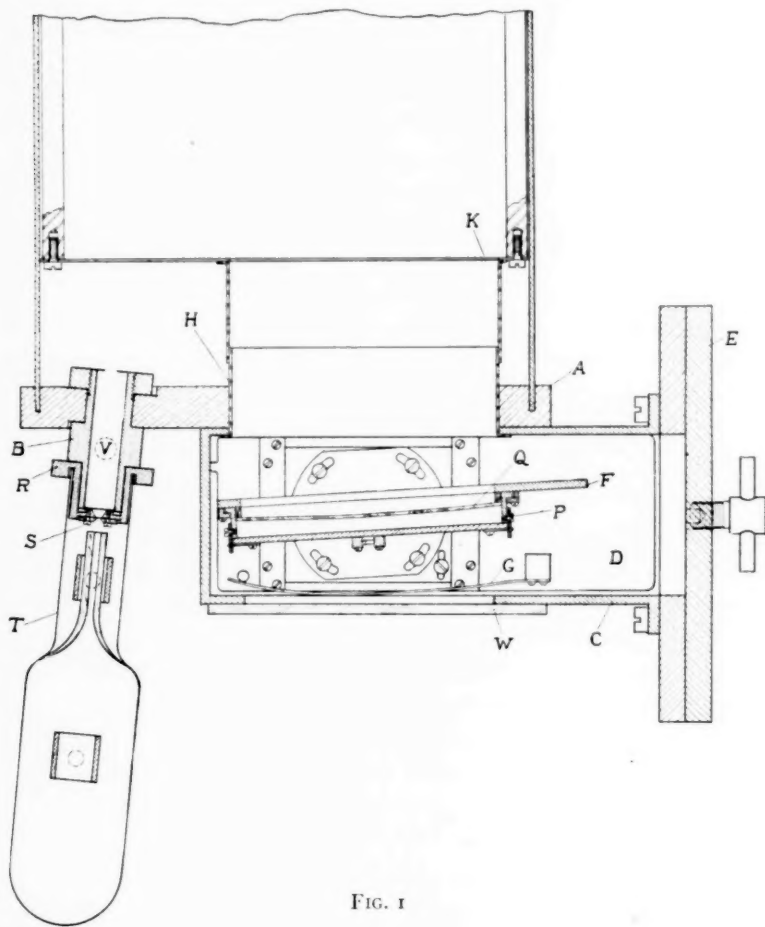


FIG. 1

vertical axis, and it is also capable of motion in a direction toward the grating or away from it. These arrangements are designed to facilitate the last stages of the focusing operations.

The photographic plate is 8 cm long and  $2\frac{1}{2}$  cm wide. A device, very similar in principle to the arrangement used in my earliest

spectroscope, is operated by an electromagnet just above *C*; by its means the plate-holder may be allowed to fall under gravity in its ways by successive equal steps; thus a number of exposures may be made on the same photographic plate.

The arrangements of the diaphragms in a spectroscope of this kind is of considerable importance. Light from the grating reaches the photographic plate through a slot 2 mm wide and 7.8 cm long in *F*. There is a corresponding slot in the plate *A* which is furnished in turn with a narrow, rectangular, boxlike sleeve, *H*, which makes connection through a second similar sleeve with a system of diaphragms, *K*, extending the length of the instrument.

The slit *S* has a height of 0.16 mm; its width is usually about 0.04 mm. It can be removed from the tube *B* for the purpose of adjustment by melting the Khotinski cement which is employed to render the joint between *B* and the collar *R* on which the slit is mounted air-tight. The discharge tube *T* is fastened to *R* with cement.

The arrangement of the pumping system is important; the outlet tube *V* is destined to remove the gas which comes through the slit from the discharge tube. The body of the spectroscope is exhausted by means of two tubes, about 1.4 cm diameter, placed symmetrically with respect to the middle point of the instrument.

The exhaustion is effected by a mercury diffusion pump backed by the Trimount vacuum pump, which I have long employed; a liquid-air trap and two drying tubes are placed between the pumping system and the spectroscope.

It is interesting to note that the process of washing with hydrogen, which I used to think all important, now seems quite unnecessary.

Except when the arrangement due to Paschen was employed, the form of the discharge tube varied little from the type shown in Figure 1 (pyrex glass was always used). The electrodes were either of aluminum or of tungsten. It is essential in order to reduce the absorption within the tube to a minimum that the end of the capillary be brought as close to the slit as possible. In all cases the discharge tube was connected to the pumps by a separate system.

Several sources have furnished the helium which has been employed, but the most important results of this research have

proved to be independent of the origin of the gas; the last lot was obtained from the United States Navy Department. After preliminary purification by chemical means the helium was passed in succession through two charcoal tubes in liquid air, then through a U tube in liquid air, and sometimes through a third charcoal tube before it found its way into the discharge tube. Tested for purity before it reached its final destination, the gas appeared satisfactory, but when it was examined in the discharge tube *T*, a trace of hydrogen was usually present, the impurity probably coming from the material of the spectroscope. This was true of the experiments tried with the 50-cm instrument as well as with the work done with the spectroscope just described. Even when *H $\alpha$*  and *H $\beta$*  were entirely invisible the presence of 1215.7, and more particularly of the second member of the series at 1025.8, gave evidence that hydrogen was not entirely absent.

It is fundamental to the nature of the experiment that a flow of gas should be maintained through the discharge tube during the time of the exposure. This flow was regulated by a fine capillary interposed between the source of the gas and the discharge. The pressure in the tube could not be directly measured, but judged by the appearance of the discharge it was usually of the order of about 1 mm. It varied somewhat during the course of any one experiment.

A continuous current was generally employed. It was furnished from a transformer taking about 8 amperes in the primary, run on the 110 A.C. circuit; kenetrons and a capacity suitably placed completed the arrangement. The current through the tube had a value of from 15 to 20 milliamperes. As the capillaries ordinarily had a diameter of about  $2\frac{1}{2}$  mm the current density was in the neighborhood of 4 milliamperes per square millimeter. When a disruptive discharge was required, a condenser and a gap of perhaps 12 mm were connected with the discharge tube in the usual way.

Schumann plates, either made in the laboratory or obtained from Hilger, were used. The time of exposure with one of Professor Wood's new gratings did not exceed one hour.

I have called attention elsewhere to the method of Duclaux and Jeantet<sup>1</sup> by which a common photographic plate may be sensitized

<sup>1</sup> *Science*, 58, 48, 1923.

for the extreme ultra-violet by the application of a thin coating of oil. The idea is of considerable utility, but, as the results are not equal to those obtained with a good Schumann plate, the process has not been employed in this research.

A report of earlier work with a grating of but 20-cm radius appeared some time ago.<sup>1</sup> In the present research two gratings of short focus have been used. The first of 50 cm was ruled at Johns Hopkins in 1914 with a total of 26,800 lines on a surface about  $4 \times 4.6$  cm.

TABLE I

HELIUM <sup>+</sup>					
$\nu = 4N\left(1 - \frac{1}{m^2}\right)$			$\nu = 4N\left(1 - \frac{1}{m^2}\right)$		
$\lambda$ Obs.	$\lambda$ Calc.	$m$	$\lambda$ Obs.	$\lambda$ Calc.	$m$
1640.4 (5)	1640.49	3	303.6 (2)	303.7	2
1215.2 ?	1215.18	4	256.3 (1)	256.25	3
1085.2 ?	1084.98	5			

HELIUM					
Principal OS-mP OS=198,298				$\lambda$ Obs.	$\lambda$ Calc.
$\lambda$ Obs.	$\lambda$ Calc.	$m$	$\nu$	OS-1P	
584.40 (10)		1	171,115	591.56 (3)	591.45
537.12 (7)	537.08	2	186,178		
522.21 (5)	522.26	3	191,493	OS-1S (?)	
515.65 (4)	515.67	4	193,930	600.3 ± .6 (6)	601.44
512.09 (3)	512.15	5	195,278		
510.05 (2)	510.05	6	196,059		
508.59 (1)	508.69	7	196,622		

In connection with this instrument it is interesting to note that the spectra in the extreme ultra-violet having become faint through an obstinate tarnish, an attempt was made to rejuvenate the surface by cathode spattering. Platinum was tried at first with some success, but after a time the film appeared to lose contact with the speculum, with the result that the last state of the grating was worse than the first. Spattering experiments with silicon have been

<sup>1</sup> Fricke and Lyman, *Philosophical Magazine*, 41, 814, 1921.

attempted with this and with other gratings, and have produced some improvement in ultra-violet reflecting power. The process looks promising and is well worth further investigation.

The second grating has a radius of 51 cm and a surface of 5 cm by 3.3 cm; it was ruled at the University of Chicago in 1921. With it the results which form the subject of the preliminary report were obtained. Within the last four months three gratings from Professor Wood have been used; they are all of 1-meter radius with a surface of about  $7.8 \times 3$  cm. The spectrogram, which is produced in Plate I, was made with one of them; and with two the first members of the extreme enhanced series at  $\lambda$  303 and  $\lambda$  256 were obtained. They are all lightly ruled and give spectra of great brilliancy in the extreme ultra-violet. It is interesting to note that in spite of a light path of 2 meters lines of the very shortest wave-lengths may be recorded; a high vacuum in the body of the spectrograph, and, above all, a grating surface free from tarnish are the factors necessary for success. This observation merely confirms Millikan's results.

#### DISCUSSION OF RESULTS

The wave-lengths of the members of the *OS*-*mP* series and of the other lines in their immediate neighborhood are given to two places of decimals, a proceeding justified by the fact that the first three members of the series were measured in the second order, taking the hydrogen line 1215.68 and the two succeeding lines of the same series as standards.

The way in which the identity of the line at  $\lambda$  1640.4 was established has been already explained. The existence of the second enhanced series really rests on the identification of this first term, for the second member, which should occur at  $\lambda$  1215.18, is almost completely obscured by the strong hydrogen line at 1215.68. This misfortune is partly to be attributed to the broad character of the lines produced by a disruptive discharge, but is chiefly due to the fact that with this type of violent excitation the ultra-violet spectrum produced by a mere trace of hydrogen is considerably stronger than the spark spectrum of the helium itself. As to the third member, which should be found at  $\lambda$  1084.9, it can only be said that a trace of it has been observed under conditions most favorable to

the purity of the gas, that is to say, by the use of the type of discharge tube recommended by Paschen and with a continuous current. As I have already pointed out, the line which is generally identified with this helium radiation is probably due to some impurity, nitrogen perhaps. Millikan gives a line at  $\lambda$  1085.2 which he attributes to this gas.

As has been already mentioned, the first ultra-violet series in the spark spectrum,  $4N(1 - 1/m^2)$ , is represented by two members at  $\lambda$  303.6 and  $\lambda$  256.3. They have been obtained with two gratings, both from the Johns Hopkins engine, but ruled under very different conditions. The first showed false spectra, but the second, having been ruled after Professor Wood had made a fundamental change in the driving mechanism of the engine, was quite free from Lyman ghosts.

The rest of the table of wave-lengths is best explained by consulting Plate I. In this illustration it must be remembered that the upper spectrum is obtained with a disruptive discharge and the lower with a continuous current; the conditions for purity are therefore less favorable in the first than in the second case. The time of exposure was the same in both. The difference of intensities is characteristic, but too much weight must not be placed on this particular example, since it is impossible to be sure that the two photographic plates on which the exposures were made were equally sensitive in the region in question. The lower spectrum is intended chiefly to illustrate the region near  $\lambda$  584; the reproduction has been cut off at or near  $\lambda$  530. The original negative shows four more members of the  $OS - mP$  series.

The almost bandlike character of the line at 600.3 is very evident as well as its intensity relation to the rest of the spectrum. It is a most persistent radiation, always visible when the exposure is of sufficient length to show the series line at  $\lambda$  537.12. The region covered by it extends from  $\lambda$  599.9 to  $\lambda$  601.3, and thus does not quite include the calculated position  $OS - 1S$ .

The interesting line at  $\lambda$  591.5 is quite clearly seen in the lower spectrum. The fact that it increases in strength with the increase in purity of the gas is an argument for ascribing it to helium. There is some question as to the origin of the line at  $\lambda$  587.5; it may perhaps be due to an impurity. However, it is very interesting to note in

this connection that an excellent spectrogram of the spark spectrum of oxygen, most kindly furnished me by Dr. J. J. Hopfield, shows no lines at either  $\lambda 587$  or  $\lambda 591$ , though there is a sharp line at  $\lambda 599$ .

The appearance of the *OS-mP* series is too striking to need much comment; even the continuous spectrum beyond its ultra-violet limit is fairly well shown. There is one point, however, which is so curious that attention must be directed to it even at the risk of repetition: The higher members of the series owe their strength and the continuous spectrum owes its existence to the fact not only that a disruptive discharge was used, but that this discharge was of a peculiar character. To obtain results similar to those shown in the upper spectrum it is essential that the discharge should run only for a second or two at a time and that about a like interval should occur between discharges. This type of excitation may limit the liberation of occluded gases, and may therefore furnish a state of higher purity than is usually obtained with the disruptive condition. When a continuous current is employed, the members of the series fall off rapidly in intensity with decreasing wave-length, and the continuous spectrum is absent.

To sum up: In the present investigation the existence of the spark series in helium,  $4N(1/2^2 - 1/m^2)$ , has been confirmed, and two members of the extreme ultra-violet series,  $4N(1 - 1/m^2)$ , have been discovered. The series *OS-mP* has been extended to a total of seven members; a new line at  $\lambda 591.6$  has been found. This last radiation is of importance as it represents a jump between energy-levels belonging respectively to the singlet and to the doublet system.

In conclusion, it is a pleasure to acknowledge my indebtedness to the skill and ingenuity of the laboratory mechanic, Mr. David Mann, who has designed and constructed my new spectroscope. I am particularly anxious to acknowledge the invaluable assistance I have received from my colleague, Professor F. A. Saunders; were it not for his modesty, which I have been unable to overcome, his name would appear with mine as joint author of this paper.

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May 5, 1924

# ON THE DETECTION OF AN EFFECT OF ROTATION DURING ECLIPSE IN THE VELOCITY OF THE BRIGHTER COMPONENT OF BETA LYRAE, AND ON THE CONSTANCY OF VELOCITY OF THIS SYSTEM

By R. A. ROSSITER

## ABSTRACT

*The spectroscopic velocity of the Beta Lyrae system.*—Elements from a series of spectrograms taken at the Allegheny Observatory in 1907 and from seven series, comprising a total of 442 spectrograms, taken at Ann Arbor in the years 1911 to 1921, show that the velocity of the center of mass of the system is constant and that elliptical motion is very well satisfied. Probably no third body exists, since no disturbance of the two-body system can be detected. The accompanying elliptic velocity curve, with radial velocities indicated by the large dots, shows how well elliptical motion is satisfied.

*The rotational effect.*—A secondary oscillation confined to the region of the velocity curve extending 1.6 days on each side of the principal minimum (or center of the eclipse of the small bright body by the large faint body) has been isolated from the orbital velocity and measured. It has been shown to be due to the velocity of rotation of the partially eclipsed smaller component, and is here termed the rotational effect. In Beta Lyrae it has a total range of 26 kilometers. A graphical determination of this curve of rotational velocity from Shapley's light-elements of Beta Lyrae gives a duration of eclipse 40 per cent longer than the spectroscope has shown and an amplitude correspondingly too great. Further investigation of this subject is under way.

This article is a summary of a part of an investigation of the brighter component of Beta Lyrae, completed by the author in April, 1923.<sup>1</sup>

Beta Lyrae is a binary system of prolate ellipsoidal stars of low density and unequal masses, which revolve about their center of mass in a period of 12.92 days. Their longest axes are in the same straight line and their adjacent vertices are nearly in contact. The plane in which they revolve passes so near the earth that each star partially eclipses the other once during each period, and causes a light variation with two unequal minima, the principal and deepest minimum occurring when the larger but fainter star eclipses the smaller but brighter star. Each star is represented by a dark-line spectrum. A third spectrum of bright hydrogen and helium lines is present, but there is a difference of opinion as to its interpretation.

<sup>1</sup> R. A. Rossiter, a dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the University of Michigan. Unpublished at the date of this writing.

The dark lines due to the brighter but less massive component oscillate across the lines due to the more massive fainter component with a range of nearly five angstrom units. The brighter component is of class B8, the fainter of class B2p.

The investigation of the orbit of the brighter component has been based upon 442 spectrograms obtained at Ann Arbor with the one-prism spectrograph attached to the 37½-inch reflecting telescope during the interval from May, 1911, to October, 1921, and upon Professor R. H. Curtiss' measures of the 1907 plates<sup>1</sup> taken with the Mellon spectrograph at the Allegheny Observatory.

#### — THE VELOCITY OF THE CENTER OF MASS

Orbital elements have been obtained for each of the years 1907, 1911, 1912, 1913, 1914-17, 1919, 1920, and 1921. In each case the elements have been adjusted by least squares.

Table I shows the final results for the velocity of the center of mass for each year mentioned above and for the elements calculated for the total interval covered by the Ann Arbor spectrograms.

TABLE I

Year	Km per Second	Year	Km per Second
1907.....	-19.1	1919.....	-19.4
1911.....	18.7	1920.....	18.6
1912.....	18.6	1921.....	19.9
1913.....	18.6	1911-21.....	-19.0
1914-17.....	-18.2		

The values obtained for the velocity of the center of mass for the eight periods covering the interval of fourteen years are nearly constant, and so completely lack a progressive tendency in any direction or any oscillation as large as a kilometer in amplitude that I have concluded that the velocity of the center of mass of Beta Lyrae is constant. Beta Lyrae is quite certainly a simple binary system, and not a three-body system as might be suspected from the earlier and less reliable spectrographic data preceding the Allegheny and Ann Arbor measures.

Structural variations in the spectral lines might lead the investigator to suspect the existence of both a third and a fourth body in

<sup>1</sup> R. H. Curtiss, *Publications of the Allegheny Observatory*, 2, No. 11.

the system, but the evidence from radial velocity specifically denies their existence by giving a constant velocity of the center of mass and a case of elliptical motion very well satisfied. When normal places computed from the final elements were compared with the observed normal places, the largest residual was  $-3.0$  kilometers and only three of the remaining thirty-one residuals were larger than  $2.0$  kilometers.

#### THE VELOCITY CURVE

Figure 1 is the ephemeris curve computed from the final elements for the total interval covered by the Ann Arbor spectrograms.

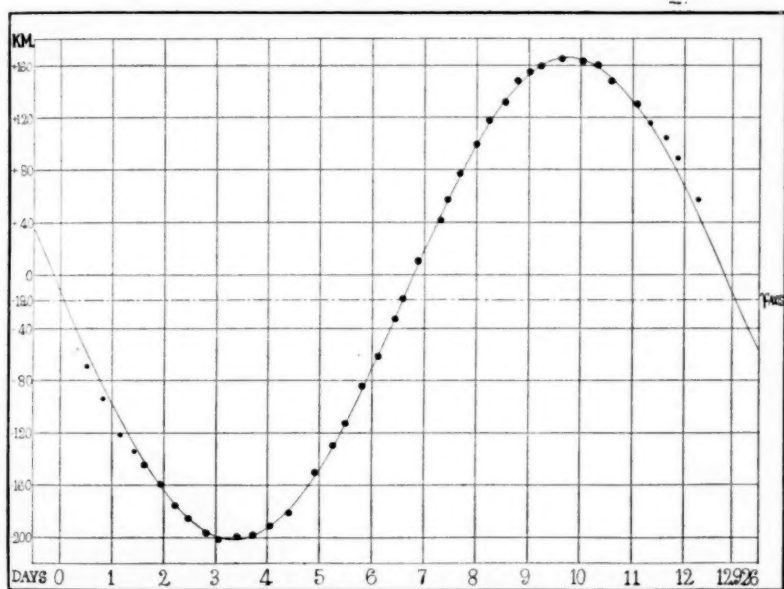


FIG. 1

The large dots are the normal places given by observation. Each place is determined by grouping from six to fifteen spectrograms. An unusually good agreement between observed motion and computed elliptical motion exists, and secondary oscillations are absent. The abnormal residuals given by the small dots will be explained in the paragraphs immediately following as a newly measured quantity here called the *rotational effect*.

## THE SECONDARY OSCILLATION NEAR PRINCIPAL MINIMUM

In the first adjustment of elements by least squares a series of unusually large residuals appeared during 1.6 days both preceding and following principal minimum (or the eclipse of the smaller bright star by the larger faint star). During the 1.6 days preceding the center of eclipse the residuals were all positive, and, during a like interval following it, they were all negative. This indicated a secondary oscillation, due to something besides orbital motion. The least-squares adjustment partially distributed this oscillation along the entire curve, but when the normal places which occur during the 1.6 days preceding and following the center of eclipse were excluded from the least-squares solution, no secondary oscillation could be detected in the remaining normal places. The large dots of Figure 1 make this evident. The secondary oscillation occurring in the residuals at eclipse time are due not to orbital motion, but to the rotation of the more luminous star about its axis.

## THE ROTATIONAL EFFECT DISCUSSED AND DEFINED

The spectrum lines of a star are symmetrically broadened by the rotation of one limb away from us and of the other with equal velocity toward us, since in one case the effect is of increasing the wave-length of the light and in the other case of decreasing it. The resulting displacement in opposite directions would broaden the lines, and equal opposite velocities would symmetrically broaden them. When the bright star is entering eclipse, one limb is gradually covered by the eclipsing star and consequently the lines from the bright star are fully broadened on one side only because of the velocity of the one wholly visible limb. When these lines are measured for determination of radial velocity, the center of density of the line will be shifted toward the broadened edge, and away from the center of the symmetrical line that would be observed if both limbs were visible. When the star is entering eclipse, the receding limb is visible and the approaching limb is covered. The measured center of the line is displaced toward the region of longer wave-lengths, and will give radial velocities too large positively. At the center of eclipse the lines are symmetrical, and are bisected where they normally should be. When the

star is emerging from eclipse, the receding limb is covered and the approaching limb is visible and consequently the measured center of the line is displaced toward the region of shorter wave-lengths. The radial velocities are then too large negatively. The small dots on the curve of Figure 1 show that the measured results agree with the foregoing discussion. The star enters eclipse at phase 11.3 days and leaves it at phase 1.6 days. In the first case the residuals are positive, and in the second case negative. This is called the *rotational effect*. It does not represent the value of the velocity of rotation at the limb of the star but represents only a fractional part of it. The magnitude of the effect depends on how much the

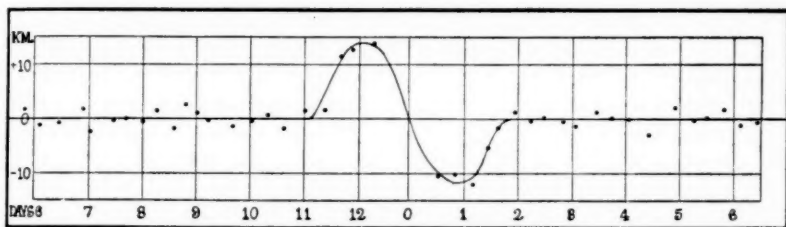


FIG. 2

light from the unbalanced visible limb is able to displace the apparent center of the line from the point where it would normally be measured. The effect starts at zero at the beginning of eclipse, reaches a maximum near the quarter phase of eclipse time, sinks to zero at the center of eclipse, reaches a negative maximum near the three-quarter phase of eclipse time, and becomes zero at the end of eclipse.

#### THE CURVE OF RESIDUALS SHOWING THE ROTATIONAL EFFECT

In Figure 2, I have put the center of eclipse at the center of the time axis of the curve, so as to exhibit more strongly the rotational effect. The residuals for each of the points of Figure 1 have been plotted as ordinates against the corresponding phase times as abscissae.

#### THE MAGNITUDE OF THE ROTATIONAL EFFECT

The rotational effect is unmistakably real and measurable. Its range in the brighter component of Beta Lyrae is 26 kilometers. It may be measurable in any eclipsing binary system, and should be

considered in orbital studies. Where the normal places affected by it have not been excluded from the least-squares adjustment of the elements, the resulting elements are somewhat distorted, and a secondary oscillation is introduced into the residuals from the ephemeris curve.

#### THE CORROBORATION OF THE EFFECT IN THE ALGOL SYSTEM

This is, I believe, the first time that this rotational effect has been isolated and measured and eliminated from the least-squares adjustment of the elements. Professor Schlesinger has suspected it in Delta Librae<sup>1</sup> and in Lambda Tauri.<sup>2</sup> Dean B. McLaughlin has measured it in the Algol system, and has found a range of about 35 kilometers. His discussion and results are published in his paper in this issue of the *Astrophysical Journal*. His studies form a very strong corroboration of the existence and measurability of this rotational effect, and indicate that other eclipsing systems should be investigated and any necessary corrections to the elements made.

From phase 12.42 days to phase 0.42 days, the lines of both components of Beta Lyrae are blended, and the resulting radial velocities are unreliable. I have, therefore, excluded them from this particular part of the discussion. They will be of use in the qualitative studies that are being undertaken.

#### GRAPHICAL DETERMINATION OF THE COMPUTED ROTATIONAL EFFECT

In order to determine whether the curve for the observed rotational effect would agree with the computed curve, I used a graphical method and employed Shapley's light-elements<sup>3</sup> to obtain the scale of the system. I computed the ellipses made by projecting the ellipsoidal bodies upon the plane tangent to the celestial sphere and perpendicular to the line of sight. Drawings were constructed to show various phases of the partial eclipse of the small body by the large body. In each case similar equal areas of the uneclipsed portion of the disk were drawn on each side of the projected axis of rotation. The remaining uneclipsed portion furnished the light

<sup>1</sup> F. Schlesinger, *Publications of the Allegheny Observatory*, 1, No. 20, 134.

<sup>2</sup> *Ibid.*, 3, No. 4, 28.

<sup>3</sup> Harlow Shapley, *Contributions of Princeton Observatory*, No. 3, pp. 71-73, 86, 87.

which widened the spectrum lines on the one side and gave the measured rotational effect. This remaining uneclipsed area was divided into narrow strips parallel to the projected axis of rotation and the area of each strip multiplied by the mean radial velocity which it possessed. It may be shown that the radial velocity of points on that portion of the surface of the body which corresponds to a strip of the projected image parallel to the projected axis of rotation depends, not on their latitude or their angle of rotation from the projection plane, but only on their apparent distance from the axis of rotation. All of the velocities in a very narrow strip are approximately the same, and the mean radial velocity of the strip is equal to the individual velocity of the midline of the strip. The summation of these products of area and velocity divided by the total uneclipsed area will give the value of this rotational effect.

#### COMPARISON OF LIGHT-DATA WITH SPECTROGRAPHIC DATA

The duration of the eclipse given by the visual or light-elements is 40 per cent greater than that given by the spectrograph, and amounts to 4.5 days instead of 3.2 days. The amplitude of the rotational effect for a uniformly illuminated disk is correspondingly larger than the measured values. For a disk completely darkened at the limb the amplitudes agree, but it is quite unlikely that there is much darkening at the limb of a bright-line star of class B8. The computed and observed curves agree satisfactorily well in form, but not in duration or amplitude. A different value of the mass ratio and of the luminosity ratio, and probably of the inclination and separation of the components, must be assumed before the computed velocities will agree with the observed spectrographic results. Further investigation is being carried out along these lines.

I wish to express my indebtedness to Professor R. H. Curtiss for the large number of spectrograms which he has made for the Beta Lyrae program, and for his kindly criticism and direction in the investigation upon which this paper is based. A word of appreciation is also due others at the Detroit Observatory who have made spectrograms and aided in various ways.

ANN ARBOR, MICH.

October 29, 1923

## SOME RESULTS OF A SPECTROGRAPHIC STUDY OF THE ALGOL SYSTEM

By DEAN B. McLAUGHLIN

### ABSTRACT

One hundred and fifty-six plates taken in 1913, 1920, and 1923 with the one-prism spectrograph of the Detroit Observatory form the basis of this discussion.

*Short- and long-period orbits.*—From plates taken within one month in 1923 orbital elements of the eclipsing system are determined. The value of  $a \sin i$  is larger than in previous determinations. With these elements as standard the velocity of the center of mass is derived for other epochs by means of simple residuals. A period of about 1.885 years and a range of 20 km are indicated for the variation of the velocity of the center of mass of the eclipsing system, substantially in agreement with the determination by R. H. Curtiss.

*Rotational effect.*—This effect due to the rotation of the brighter star during the partial eclipse, discovered by Dr. Rossiter in Beta Lyrae, is investigated in Algol and is found to have a range of 35 km as shown in the diagram. Computation with Stebbins' light-elements gives only half the observed range of the effect, on assumption that  $m_b = 2m_f$ . Investigation of this rotational effect is suggested as a means of determining dimensions of other eclipsing systems. Agreement of form of computed and observed curves serves as a check on light-elements.

*Dimensions of the eclipsing system.*—Assuming equal periods of revolution and rotation of the bright star the dimensions of the eclipsing system are calculated. The total mass is  $5.67 \odot$ . Mass ratio is  $m_b = 5.0 m_f$ . Radii of the stars are:  $r_b = 3.12 \odot$ , and  $r_f = 3.68 \odot$ . Distance between centers: 10,522,000 km. Densities:  $d_b = 0.16$ ,  $d_f = 0.02$  ( $\odot = 1$ ).

*Parallax.*—Hypothetical parallax of Algol is  $0''.031$ . Orbital motion is suggested as a possible cause of negative parallaxes of certain stars.

The present paper is intended to present some of the results of a study of the Algol system based on 156 plates taken with the single-prism spectrograph of the  $37\frac{1}{2}$ -inch reflector of the Detroit Observatory, and to some extent on the published velocities of earlier observers. Final results will appear later in the *Publications of the Detroit Observatory*.

The Ann Arbor spectrograms were taken at four epochs: 1913.73, 1920.81, 1923.24, and 1923.66, excluding two isolated pairs of plates. By far the most numerous group is that of the present season, which includes 79 plates. Of these, 39 were used for the determination of orbital elements of the eclipsing system; the remaining 40 were taken for the express purpose of investigating this effect of the star's rotation discovered by Dr. Rossiter in Beta Lyrae.

## ORBIT OF THE BRIGHT STAR

In order to prevent bias entering into the measures, the number on each of the 79 plates was covered by a strip of paper. The plates were then thoroughly shuffled, and were designated by letter combinations. They were not identified until all the measures were completed. The following preliminary elements for the orbit of the bright star were obtained from the 39 plates taken outside eclipse.

TABLE I

ЕPOCH 1923.66

$e$ .....	0.038
$T$ (after light-minimum).....	1.506 days
$\omega$ .....	$277^{\circ}5$
$K$ .....	44.1 km
$\gamma$ .....	+16.9 km
$a \sin i$ .....	1,736,000 km

Attention is called especially to the value of  $a \sin i$ . It is the largest value ever obtained for Algol, and the accuracy of the observations seems to be such as to prohibit reducing it below 1,700,000 km. We shall see presently a possible reason for this large value.

## THE LONG-PERIOD ORBIT

It is well known that the velocity of the center of mass of the eclipsing system is variable. Some years ago Professor R. H. Curtiss published a set of circular elements for the long-period orbit.<sup>1</sup> The most probable period appeared to be 1.899 years. On the basis of that period, the present determination falls on the peak of the curve. The entire 39 plates were taken within four weeks, and the change of the velocity of the center of mass during that period would be negligible at the maximum point of the curve. *Hence the elements above are unaffected by variation of the velocity of the center of mass during the period of observation.* All previous determinations were made at such points on the curve and extended over such long periods of time that considerable change of velocity of the center of mass occurred between the taking of the first and last plates. I have examined the Allegheny observations<sup>2</sup> and find that more than half the discrepancy between the Allegheny velocity range and mine

<sup>1</sup> *Astrophysical Journal*, 28, 150-161, 1908.

<sup>2</sup> Schlesinger and Curtiss, *Publications of the Allegheny Observatory*, 1, 25-32, 1910.

can be removed by applying corrections for change in the velocity of the center of mass to the individual plates. A rediscussion of the Pulkova and Allegheny velocities is being undertaken.

Using the orbital elements above as standard, the velocity of the center of mass of the eclipsing system was determined for 1913.73, 1920.81, and 1923.24 by taking simple residuals. All these plates serve to strengthen the short-period curve of 1923.66. The 1920 plates especially can be brought to lie very closely along it.

With the velocities thus obtained, in addition to those found by the earlier observers, various periods were tried. It was found that a period midway between those of R. H. Curtiss and F. Schlesinger<sup>1</sup> gave the best results. Periods of 1.55, 1.624, 1.69, 1.71, 1.73, 1.88, 1.89, and 1.899 years were tried, but a value of 1.885 appeared to satisfy the observations best. The period of 1.624 which R. H. Curtiss examined as a possible, but very doubtful, alternative to 1.899 is completely disproved. J. Stebbins found that this period seemed to satisfy his photometric observations better.<sup>2</sup> At the epoch 1923.66 the maximum of the 1.899-year curve and the minimum of that of 1.624 nearly coincide. The velocity of the center of mass in 1923 discredits the latter completely. With a period of 1.69 years all the observations are satisfied fairly well except Belopolsky's of 1897, based on 22 plates, which has a residual of about  $-10$  km. This period was therefore rejected. The indications are that a period of 1.899 years is too long, and 1.874 (Schlesinger) too short. As final elements cannot be determined until the old observations have been corrected for change of velocity of center of mass, the period cannot now be settled definitely. The following preliminary elements, however, probably represent a fair approximation:

TABLE II

$E$ (epoch of minimum velocity) . . . . .	1901.85
$P$ . . . . .	1.885 years
$e$ . . . . .	0.13
$T$ (after minimum velocity) . . . . .	0.943 years
$\omega$ . . . . .	$0^\circ$
$K$ . . . . .	10.0 km
$\gamma$ . . . . .	+5.7 km
$a \sin i$ . . . . .	93,000,000 km

<sup>1</sup> F. Schlesinger, *Publications of the American Astronomical Society*, 2, 128, 1912.

<sup>2</sup> *Ibid.*, 4, 331, 1922.

## THE ROTATIONAL EFFECT

In the fall of 1922, Dr. R. A. Rossiter, of this observatory, in plotting the residuals of the normal places used in his investigation of Beta Lyrae, found large positive residuals just preceding the middle of the principal eclipse, and large negative ones immediately following it. He discusses his results in a paper in this issue of the *Astrophysical Journal*. This he interpreted as an effect of the rotation of the partially eclipsed star. A few months later I completed the reductions of the measures of the 1913 and 1920 plates of Algol. Apparently by accident 9 of the 20 plates taken in 1913 were made during eclipse, just after minimum light. Without exception these plates lay far below the Allegheny curve when it was made to pass through the observations outside eclipse. Owing to the poor distribution of the plates it was impossible to decide at the time whether this discrepancy was due to the rotational effect or to a change of the orbital elements since 1907. At the time the  $37\frac{1}{2}$ -inch reflector was being overhauled, and observations could not be begun until March, 1923, when Algol was far west. Twenty plates were then obtained, covering the curve fairly well, but attempts to observe during eclipse were frustrated by the weather.

In August observations were begun again and extended from August 14 to September 10. Clear weather, a carefully planned program, and special care to obtain plates of the best density resulted in material for the determination of an excellent velocity curve and a preliminary curve for the rotational effect. Further plates were taken on September 14 and October 7. To these were applied corrections for change of velocity of center of mass on the basis of the elements above of the long-period orbit.

The curve for the rotational effect was determined by means of the residuals of 40 plates taken during eclipse, these residuals being determined on the basis of the orbital elements derived from the 39 plates outside eclipse. The residuals before minimum are all positive, those after minimum are without exception negative, in accordance with theory. The figure shows conclusively the reality of the rotational effect. The curve drawn is not the observed but the computed curve. For the negative branch of the curve the difference between the forms of the computed and observed

curves is negligible. Observations to cover better the positive branch will be obtained as soon as possible.

The rotational effect is very easily computed graphically when we know the light-elements of the eclipsing system. In the case of a star so thoroughly studied as Algol it might be expected that an accurate computation of the effect could be carried out. On the basis of Stebbins' elements for 1920,<sup>1</sup> I computed the curve for the rotational effect. I assumed a mass ratio  $m_b = 2m_f$ , and equal periods

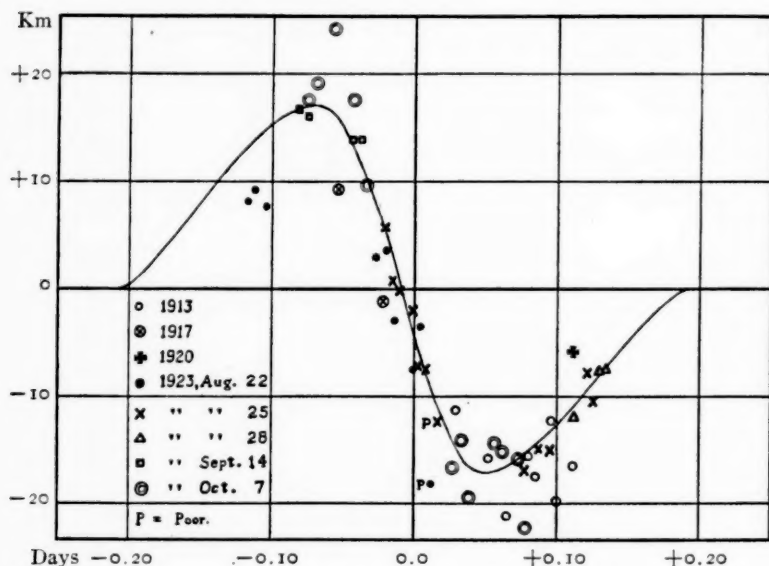


FIG. 1.—Curve of the rotational effect in Algol

of rotation and revolution of the bright star, on which assumptions the equatorial velocity of rotation of the bright star multiplied by  $\sin i$  is 27.3 km per sec. It was found that an agreement between observation and theory could be obtained only by multiplying each ordinate of the computed curve by 2.0. When this was done the agreement was excellent, and it is this curve which is shown in the figure. I have not shown normal places in order to avoid incumbering the diagram with further symbols, but any reader who so wishes can easily form them. In so doing we should give greatest weight

<sup>1</sup> *Astrophysical Journal*, 53, 116, 1921.

to the 1923 plates, as they alone are certainly homogeneous as regards computed phase. The curve has been moved along the time axis in the negative direction 0.01 day to secure better agreement with observation. Evidently the minima actually occurred about 0.01 day before the computed time. It is interesting to observe that there are still such important uncertainties in the predicted times of minimum of this bright variable star.

#### INTERPRETATION OF THE RANGE IN ROTATIONAL EFFECT

The first curve computed for the rotational effect was found to have a range of only 17.7 km, on the assumptions that the mass of the bright star is twice that of the faint one and that the periods of revolution and rotation are equal. The observed range of 35 km or possibly more must mean then that one or both of these assumptions are erroneous; either the bright star is rotating in a shorter period than 2.8673 days or the scale of the system is greater than supposed in the ratio of 2.0 to 1. The tidal action in the Algol system must certainly have prevented a difference between the orbital and rotational periods. Additional confirmation of this view is found in Stebbins' detection of ellipsoidal figure in the bright star,<sup>1</sup> as shown by slight curvature of the light-curve during maximum. There is, of course, the alternative view that this represents a tremendous tide on the surface of the star, which may be rotating in a shorter period than 2.8673 days, but this appears highly improbable. H. Shapley has stated<sup>2</sup> that in Algol the periods of rotation and revolution are "almost certainly" different. He proceeds to qualify this statement, however: "Or it may be, on the other hand, that it is merely a matter of accurate photometry to show the existence of the curved maxima."

#### DIMENSIONS OF THE ECLIPSING SYSTEM

I believe, therefore, that we are compelled to take the view that the assumption of a 2 to 1 mass ratio makes the Algol eclipsing system too small. The following elements by Stebbins<sup>3</sup> define the

<sup>1</sup> *Ibid.*, p. 114.

<sup>2</sup> *Contributions from the Princeton University Observatory*, No. 3, 114, 1915.

<sup>3</sup> *Astrophysical Journal*, 53, 116, 1921.

relative dimensions of the system. The radius of the relative orbit is taken as unity.

Radius of bright body..... $r_b$ .....	0.207
Radius of faint body..... $r_f$ .....	0.244
Cosine of inclination..... $\cos i$ .....	0.142

Using the value of  $a \sin i$  obtained above, we have, on the assumption of 2 to 1 mass ratio, radius of relative orbit,

$$a_b + a_f = 3a = 5,261,000 \text{ km.}$$

But the scale of the system must be increased in the ratio of 2.0 to 1, to account for the observed limb velocity, hence

$$a_b + a_f = 10,522,000 \text{ km} = 6.0 a,$$

and therefore the mass ratio is  $m_b = 5.0 m_f$ . The following table gives the resulting values for the radii, masses, and densities of the stars in terms of the sun, determined on the two assumptions  $m_b = 5.0 m_f$  and  $m_b = 2 m_f$ . The masses were computed from the formula

$$m_b + m_f = \frac{4}{10^{20}} \frac{(a_b + a_f)^3}{P^2}.$$

TABLE III  
DIMENSIONS OF THE ALGOL ECLIPSING SYSTEM

		$m_b = 5.0 m_f$	$m_b = 2 m_f$
Radius of bright body....	$r_b$	3.12 $\odot$	1.56 $\odot$
Radius of faint body.....	$r_f$	3.68	1.84
Mass of bright body.....	$m_b$	4.72	0.47
Mass of faint body.....	$m_f$	0.95	0.24
Density of bright body....	$d_b$	0.16	0.12
Density of faint body.....	$d_f$	0.02	0.04
Distance between centers..	$a_b + a_f$	10,522,000 km	5,261,000 km

Expressed in kilometers, the radii of the bright and faint stars are respectively 2,178,000 and 2,567,000 in the case of a large system, or 1,089,000 and 1,284,000 in the smaller system.

The larger system appears in every respect the more probable. In the case of the smaller system we have surprisingly small masses, especially in view of the fact that the bright star is of class B8. The known ratio of surface brightnesses (about 17 to 1) indicates a great difference in spectral type of the two stars, and we should therefore expect a ratio of densities of 8 to 1 rather than one of 3 to 1. Apparently the faint star is on the "giant" branch of the absolute-magnitude curve, and the bright one on the "dwarf" branch, although in this particular case the terms "giant" and "dwarf" are rather misleading. Rather, we should say, that the faint star is increasing in brightness and the bright one decreasing, or possibly about at its maximum. Apparently all considerations favor a large system, with the bright star five times as massive as the faint one.

#### IMPORTANCE OF THE STUDY OF ROTATIONAL EFFECT

The lines of a spectrum are more susceptible of accurate measurement during partial eclipse than outside eclipse because of the fact that a considerable portion of the rotating star is hidden and the rotational broadening of the lines reduced.

*The investigation of this rotational effect appears to be the logical method to employ in determining the actual scale of eclipsing systems in which only one spectrum is measurable.* The light-curve gives us the relative dimensions of the system and the means of computing, on some assumption of masses, the range of the rotational effect. The *observed range* of the effect gives us the value of the one unknown that is needed, viz., the absolute scale of the system. When we consider that the true masses and densities may thus be obtained, the importance of the study of this rotational effect becomes evident.

In addition to this, the agreement of form of the observed and computed curves for this rotational effect furnishes a check on the *relative dimensions* as determined from the light-curve. It may also aid in the investigation of darkening at the limbs of stellar disks. In the case of Algol there is little or no evidence of darkening.<sup>1</sup> It is unfortunate that so few eclipsing variables are bright enough to be investigated for this effect.

<sup>1</sup> J. Stebbins, *Astrophysical Journal*, 53, 117, 1921.

## THE PARALLAX OF ALGOL

We are now in a position to compute the hypothetical parallax of Algol. The hypothetical parallaxes of eclipsing binaries were computed by Shapley<sup>1</sup> on the basis of three assumptions: (1) the radius of the bright star, resulting from the assumption that the mass of the system is twice that of the sun; (2) the mean value for the mass,  $2\mu$ , of a system of the particular spectral type; (3) the surface brightness,  $J_b$ , of the bright star, reckoned in stellar magnitudes in excess of the surface brightness of the sun. It is now possible to use the radius obtained on the basis of an assumption which amounts almost to a certainty, viz., the equality of the periods of rotation and revolution. From the formula

$$M = J_b - 5.0 \log r_b + 4^m.75$$

we obtain absolute magnitude of Algol =  $-0^m.28$ , and from

$$M - m_b = 5 + 5 \log \pi,$$

where  $m_b$  is the apparent magnitude of the bright star, we find the hypothetical parallax of Algol =  $0''.031$ . The value of  $J_b$  used is  $-2^m.56$ , corresponding to spectral class B8 as given by Shapley<sup>1</sup> after Russell.

No reliable trigonometric parallax of Algol has yet been obtained, due to the fact that the eclipsing system is moving in an orbit about two-thirds as large as its apparent parallactic orbit in a period of less than two years. It would probably be well worth while to re-determine the parallax by introducing into the equations of condition two more unknowns, one expressing the eccentricity of the apparent orbit, the other its orientation. It is obvious that the semimajor axis of the apparent orbit is equal to about  $\frac{2}{3}\pi$ . In this connection it may not be out of place to suggest that some negative stellar parallaxes may be a consequence of orbital motion in a short period. Cases of large probable errors and of great disagreement of spectroscopic and trigonometric parallaxes might prove extremely interesting.

<sup>1</sup> *Op. cit.*, No. 3, p. 117, 1915.

It is the intention of the writer to study this rotational effect in connection with the determination of the absolute dimensions of the eclipsing systems  $\lambda$  Tauri,  $\delta$  Librae, RZ Cassiopeiae, and VV Orionis.

I am greatly indebted to Professor R. H. Curtiss, who supervised this investigation and furnished helpful suggestions and criticism as well as the opportunity to take many of the plates upon which this study is based. I am also indebted to various members of the observatory staff, who took the earlier plates.

DETROIT OBSERVATORY

ANN ARBOR, MICH.

October 31, 1923

## PRESSURE AND CIRCULATION IN THE REVERSING-LAYER OF THE SUN'S ATMOSPHERE<sup>1</sup>

By CHARLES E. ST. JOHN AND HAROLD D. BABCOCK

### ABSTRACT

*Pressure in the reversing layer.*—New values of over two hundred solar wave-lengths measured at Mount Wilson by two independent methods, the arc wave-lengths adopted by the International Union, and unpublished data on pressure displacements thought to be practically free from pole effect, furnish a basis for a determination of the pressure in the solar reversing-layer. *The method is purely differential:* pressure in atmospheres =  $(\Delta\lambda_2 - \Delta\lambda_1) \div (c_2 - c_1)$  where the  $c$ 's are pressure coefficients and the  $\Delta\lambda$ 's are differences in wave-length between the sun and the vacuum arc. *A suitable choice of lines* eliminated effects dependent on level in the sun's atmosphere, on motion in the line of sight, and on relativity. *A pressure of  $0.13 \pm 0.06$  atm.* was found for the lower few hundred kilometers of the reversing-layer. Heights, behavior in the vicinity of spots, vertical convection, rotation, pressure, and data derived from flash spectra are discussed and correlated in a suggested *scheme* of the sun's atmosphere.

The data accumulated at the Mount Wilson Observatory are now sufficient for an attack on the pressure in the sun's atmosphere with considerable confidence in the outcome. The earlier estimates by Jewell, Mohler, and Humphreys<sup>2</sup> and by Fabry and Buisson<sup>3</sup> were based upon the absolute differences between solar and arc wave-lengths. On the assumption that the solar atmosphere is comparatively quiescent and that displacements toward longer wave-lengths are due entirely to increase of pressure in passing from the terrestrial to the solar atmosphere, Jewell, Mohler, and Humphreys obtained from the iron lines 7 atm. for the pressure in the reversing-layer, and Fabry and Buisson 5–6 atm. from four-score iron lines distributed over two regions.

The recognition of classes of lines of the same element with characteristically different pressure-coefficients makes possible a differential method. Evershed<sup>4</sup> compared Rowland's solar wave-lengths with Kayser's original wave-lengths of iron for lines "most affected" and "least affected" by pressure, and concluded that the

<sup>1</sup> Contributions from the Mount Wilson Observatory, No. 278.

<sup>2</sup> *Astrophysical Journal*, **3**, 138, 1896.

<sup>3</sup> *Comptes Rendus*, **148**, 688, 1909.

<sup>4</sup> *Kodaikanal Bulletin*, No. 18, 1909, and No. 36, 1913.

pressure in the sun did not exceed one atmosphere; but lines with great pressure-coefficients—"most affected" lines—belong to types with large pole-effect, and the early observations of the difference between sun and arc for such lines are now known to have been greatly in error.<sup>1</sup>

If two groups of lines have pressure-coefficients per atm.  $c_2$  and  $c_1$ , and if  $\Delta\lambda_2$  and  $\Delta\lambda_1$  are changes in wave-length due to an increase of pressure, then  $p$ , the added pressure in atmospheres,  $= (\Delta\lambda_2 - \Delta\lambda_1) \div (c_2 - c_1)$ . The advantages of the method for investigations like the present are that it requires only the difference of the  $\Delta\lambda$ 's and not their absolute values, that it avoids the assumption that pressure alone is the cause of the displacement, and that with suitable data it eliminates both Doppler and relativity effects. Adams<sup>2</sup> applied this method in his preliminary investigation of pressure in stellar atmospheres, using as a basis the systematically different pressure-coefficients of enhanced and arc lines. Perot<sup>3</sup> applied a similar method to the sun and, using two lines of magnesium,  $\lambda 5172$  and  $\lambda 5183$ , obtained a very low value for the pressure; Salet<sup>4</sup> similarly used a dozen lines of iron and also compared the positions of the lines as given in Rowland's table with their positions in the arc spectrum for lines of different types and found a pressure of some tenths of an atmosphere. As the wave-lengths of the magnesium lines used by Perot and the very large pressure-coefficients used by Salet and by Evershed for lines of types  $c$  and  $d$  are greatly influenced by pole-effect, and as the general tendency of pole-effect is to give fictitiously low values for the pressure in the sun, it is necessary to redetermine the pressure with homogeneous data free, as far as possible, from this disturbing influence, if a reliably definite result is to be obtained. With the elimination of pole-effect the differences between pressure-coefficients for the different types of lines become much smaller than they at first appeared to be, and this requires data in large quantity to give assurance to the findings. The methods employed to free our data from this

<sup>1</sup> *Mt. Wilson Contr.*, No. 137, p. 24; *Astrophysical Journal*, **46**, 161, 1917.

<sup>2</sup> *Mt. Wilson Contr.*, No. 50; *Astrophysical Journal*, **33**, 64, 1917.

<sup>3</sup> *Comptes Rendus*, **172**, 578, 1921.

<sup>4</sup> *Ibid.*, **174**, 151, 1922.

insidious source of error are described in our two papers bearing directly on the subject.<sup>1</sup>

The data for the present discussion are the solar wave-lengths on the international system, measured at Mount Wilson Observatory by grating spectrographs and interferometers, and the arc wave-lengths adopted at the Rome meeting of the International Astronomical Union,<sup>2</sup> supplemented by additional lines from the lists published by St. John and Babcock.<sup>3</sup> The pressure-coefficients used in reducing arc wave-lengths at atmospheric pressure to the wave-lengths for the arc in vacuum are revised values from laboratory investigations at Mount Wilson Observatory not yet ready for publication in full.

For the purpose of the present investigation it is necessary to eliminate as far as possible effects depending upon wave-length and upon differences of level in the solar atmosphere. By using sets of lines which have approximately the same mean intensity and among which the dispersion in intensity is small, great differences of level with consequently unequal convection are avoided. By further limiting the constitution of the sets to lines in approximately the same spectral region, both Doppler and relativity effects are rendered negligible. Evidence that anomalous refraction plays no appreciable rôle in the spectrum of the center of the sun has been given in previous contributions.<sup>4</sup>

Mean results for eleven sets of lines fulfilling the necessary conditions are given in Table I. These were formed by selecting ten neighboring lines having large pressure-coefficients and, closely surrounding and intermingled among them, an equal number of lines with smaller coefficients; since lines having large coefficients are relatively infrequent, the groups are necessarily restricted. Lines with smaller coefficients are, however, more numerous, and well-balanced groups can usually be found; in only two instances does the number of lines fall below ten.

<sup>1</sup> *Mt. Wilson Contr.*, Nos. 106 and 137; *Astrophysical Journal*, **42**, 231, 1915, and **46**, 138, 1917.

<sup>2</sup> *Transactions International Astronomical Union*, **1**, 35, 1922.

<sup>3</sup> *Mt. Wilson Contr.*, No. 202, 1921; *Astrophysical Journal*, **53**, 260, 1921.

<sup>4</sup> *Mt. Wilson Contr.*, Nos. 93 and 123; *Astrophysical Journal*, **41**, 28, 1915, and **44**, 311, 1916.

The first column of the table shows the composition of the sets, the upper group in each set having the larger pressure-coefficient. With an appreciable pressure in the reversing-layer, prevailing

TABLE I  
PRESSURE IN THE SUN'S REVERSING LAYER

Group	No. of Lines	Mean Solar Intensity	Mean Wave-Length	Sun minus Vacuum	Pressure-Coefficients	Pressure in Sun
<i>b</i> .....	10	13.8	3719	+0.0107 A	0.0019 A	-0.089
<i>a</i> .....	10	14.	3759	+0.0115	0.0010	
<i>d</i> .....	10	4.6	4170	+0.0061	0.0040	-0.26
<i>b</i> .....	10	5.	4158	+0.0066	0.0021	
<i>b</i> .....	10	6.	4181	+0.0101	0.0021	-0.22
<i>a</i> .....	10	8.3	4191	+0.0103	0.0012	
<i>d</i> .....	10	4.8	4239	+0.0077	0.0043	+0.54
<i>b</i> .....	10	4.	4237	+0.0065	0.0021	
<i>c</i> .....	6	5.	4458	+0.0080	0.0036	+0.07
<i>b</i> .....	7	7.	4442	+0.0088	0.0022	
<i>b</i> .....	10	3.8	4592	+0.0061	0.0023	+0.75
<i>a</i> .....	10	3.5	4628	+0.0055	0.0015	
<i>d</i> .....	10	3.7	4615	+0.0051	0.0048	-0.09
<i>a</i> .....	10	3.	4632	+0.0054	0.0015	
<i>b</i> .....	10	3.3	4716	+0.0057	0.0023	+0.59
<i>a</i> .....	10	3.9	4766	+0.0053	0.0016	
<i>c</i> .....	10	5.4	4904	+0.0062	0.0052	+0.29
<i>b</i> .....	10	2.8	4922	+0.0054	0.0024	
<i>b</i> .....	10	4.1	5542	+0.0078	0.0027	-1.00
<i>a</i> .....	10	4.8	5431	+0.0084	0.0021	
<i>Mn</i> * .....	7	4.7	4773	+0.0070	0.0054	+0.33
<i>b</i> .....	7	3.4	4762	+0.0060	0.0024	
Weighted mean ..						+0.13 ± 0.06 atm.

\* Monk, *Astrophysical Journal*, 57, 222, 1923.

positive values for the deduced pressures would be expected. From the seventh column it will be seen, however, that there is a nearly equal division between positive and negative signs. On applying the usual rules for the propagation of errors, the weighted mean is

$0.13 \pm 0.06$  atm. This result refers to the lower few hundred kilometers of the atmosphere, and indicates that the pressure in the reversing-layer is at most but a small fraction of an atmosphere. Pointing in the same direction is the remarkable sharpness in the sun of lines that in the laboratory become broad and diffuse under moderate pressure.

Low pressures in the solar atmosphere have been deduced from the theory of ionization. Saha<sup>1</sup> estimates that the ionization of calcium in the sun would be complete at a pressure of  $10^{-4}$  atm. The level at which this pressure occurs would be the upper limit of normal calcium registered spectroscopically by  $\lambda 4227$ , its fundamental line. In a paper by one of the present writers it was calculated from the increased intensity of enhanced lines over faculae that the pressure at medium levels is of the order  $10^{-1}$  to  $10^{-2}$  atm.<sup>2</sup> Russell<sup>3</sup> finds that for the alkali metals and earths the increased intensity of the Fraunhofer lines in passing from the sun  $6000^\circ$  K. to spots  $4000^\circ$  K. is accounted for by the percentage of ionization occurring at pressures not greater than  $10^{-2}$  atm. in the region of absorption.

In a recent paper by Stewart on "The Opacity of an Ionized Gas,"<sup>4</sup> the opacity of ionized sodium due to absorption by free electrons is estimated for pressures from 1 to  $10^{-4}$  atm. He concludes from these results "that at the depth where the pressure is as great as 0.01 atmosphere the opacity is sufficient to cut off light from lower levels. This figure, then, is indicated as an upper limit to the pressure in the visible regions in the solar atmosphere."

R. H. Fowler and E. A. Milne, on the assumption that the vapor of  $\text{Ca}^+$  is in equilibrium under radiation pressure and gravitation, find a mean pressure of the order of  $10^{-13}$  atm. for the upper 5000 km of this vapor. Its uppermost limit determined by the H and K lines is, according to Mitchell, 14,000 km.<sup>5</sup>

<sup>1</sup> *Philosophical Magazine*, **40**, 809, 1920.

<sup>2</sup> St. John, *Contr. Jefferson Physical Laboratory*, **15**, 1921.

<sup>3</sup> Mt. Wilson *Contr.*, No. 225, 1922; *Astrophysical Journal*, **55**, 134, 1922.

<sup>4</sup> *Physical Review*, **22**, 324, 1923.

<sup>5</sup> Estimates of absolute level are based upon Mitchell's measures of eclipse spectra (*Astrophysical Journal*, **38**, 407, 1913).

Our direct observations of a pressure of  $10^{-1}$  atm. refer to relatively low levels. The upper limit of normal calcium for which Saha estimates a pressure of  $10^{-4}$  atm. is 5000 km above the visible surface. Russell's and Stewart's value of  $10^{-2}$  atm. refers more nearly to the levels to which our observations correspond. The relative levels of normal calcium,  $\lambda_{4227}$ , of the D lines of sodium and the H and K lines of ionized calcium are shown by their velocities of inflow into spots, 0.13, 0.38, and 3.78 km/sec., respectively, which increase *pari passu* with elevation.<sup>1</sup> According to Russell,<sup>2</sup> the percentage of sodium remaining un-ionized in the reversing-layer is 1.1 and of calcium 6.8, and the ratio of the two percentages is independent of pressure. Other considerations being the same, the higher percentage of neutral calcium implies that it rises to higher levels than the neutral sodium, and hence the level of  $\lambda_{4227}$ , the ultimate line of normal calcium, should be above that of D<sub>2</sub>, the ultimate line of normal sodium, but the evidence from observation points to the opposite order in level. The percentage of ionization is, however, only one factor; another is the proportional amount in the sun. For this the only basis of judgment is the relative abundance of sodium and calcium in the earth's crust. The percentage by weight given by Clarke and Washington<sup>3</sup> is 2.61 for sodium and 3.37 for calcium; but, as H. H. Plaskett<sup>4</sup> remarks, we are interested here in the percentage of atoms, which is 2.03 for sodium and 1.50 for calcium. This requires, for equal numbers of atoms, a mass ratio of sodium to calcium in the sun 4.5 times the ratio of abundance in the earth's crust. Other considerations are the atomic weights, which favor the higher extension of the sodium, and the relative ability of the sodium and calcium atoms to absorb their respective energy quanta and to return to their receptive states. The D lines dominate the spectrum of neutral sodium to a greater degree than  $\lambda_{4227}$  dominates the spectrum of neutral calcium. The calcium spectrum contains other powerful lines, while the sodium spectrum contains no other lines

<sup>1</sup> *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 322, 1913.

<sup>2</sup> *Op. cit.*, p. 134.

<sup>3</sup> *Proceedings National Academy Sciences*, **8**, 119, 1922.

<sup>4</sup> *Publications Dominion Astrophysical Observatory*, **1**, 375, 1922.

comparable with the D lines in intensity. The selective mechanism of the sodium atoms seems to favor in higher measure the preferential distribution of the absorbed energy to its dominating lines. The enormous absorbing power of sodium vapor has long impressed observers, but no quantitative comparison of sodium and calcium is known to the writers. Further evidence of the higher levels of the D lines is found in their more frequent occurrence in the chromospheric spectrum, 50 to 3 according to Young,<sup>1</sup> and their occasional appearance even in prominences. The relative behavior of the D lines and  $\lambda 4227$  in the sun seems not to be explained on the basis of ionization potentials alone, nor by any probable difference in their relative abundance.

The terms "chromosphere" and "reversing-layer" have unfortunately rather indefinite meanings in the literature of the sun. Deslandres suggests that the whole solar envelope outside of the photosphere be considered the atmosphere of the sun, and it is in this sense that the term "atmosphere" is used in the present paper. R. H. Fowler and Milne suppose that the reversing-layer is the region in which equilibrium under an appreciable pressure-gradient gives place to low-pressure equilibrium under radiation-pressure. The division between reversing-layer and chromosphere would be the level at which first-step ionization is practically complete, that is, the upper level of neutral calcium.<sup>2</sup> This corresponds closely to the level at which the movements of the absorbing vapors around spots change from outflow to inflow<sup>3</sup> and implies a definite physical state for the chromosphere, the complete ionization of the metallic vapors.

In the study of phenomena of solar and stellar atmospheres, questions of pressure, levels, and currents come necessarily into consideration. In the accompanying chart (Fig. 1) an attempt is made to sketch in tentative outlines a scheme of the character and relative distribution of these factors in the sun's atmosphere. In the first column under "Composition," a few elements characteristic of certain levels are arranged in order of the magnitude and direction of

<sup>1</sup> Scheiner, *Astronomical Spectroscopy*, translated by Frost, p. 184, 1894.

<sup>2</sup> *Monthly Notices, R.A.S.*, **83**, 403, 1923.

<sup>3</sup> *Mt. Wilson Contr.*, No. 69, p. 21; *Astrophysical Journal*, **37**, 342, 1913.

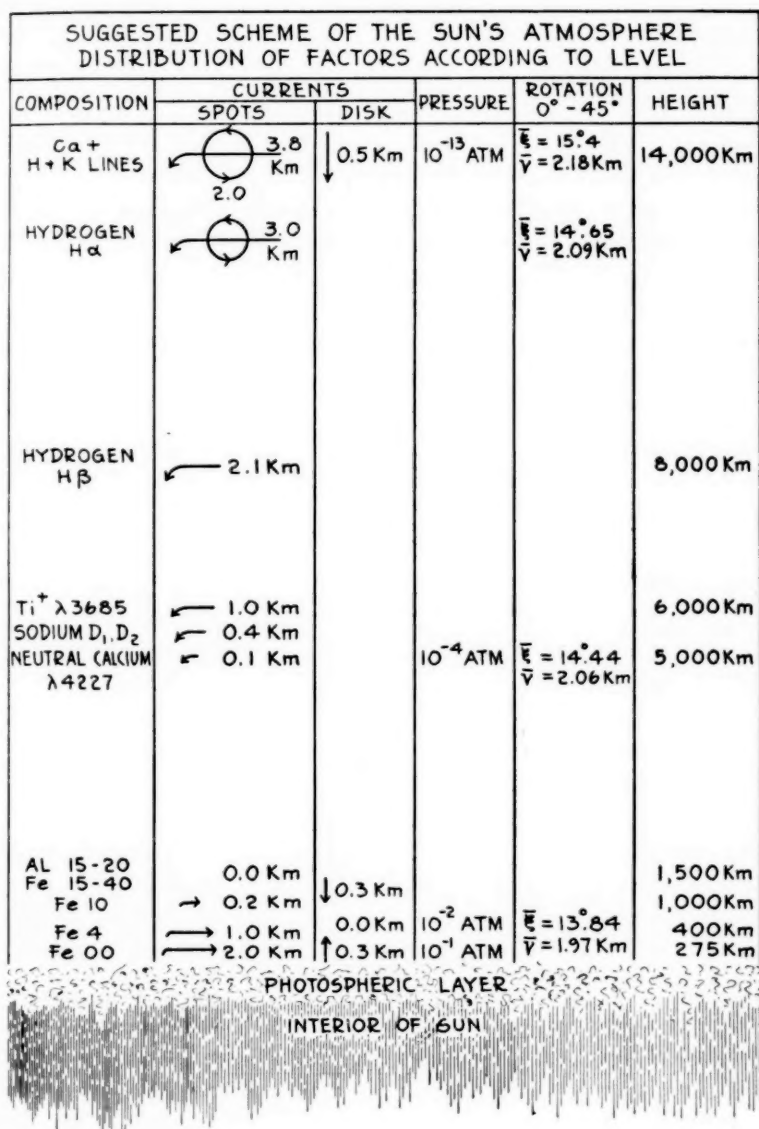


FIG. 1

the currents in the immediate neighborhood of spot vortices, indicated in the second column by the length and direction of the arrows. For  $Ca^+$  the tangential component of the vortical whirl, indicated by the circle, is a directly measured velocity; for  $H\alpha$  it is based upon the lines of flow on spectroheliograms. At the highest observed elevation above the visible surface, ionized calcium is flowing into spots with a velocity of nearly 4 km/sec. At successively lower elevations hydrogen, ionized titanium, and neutral sodium and calcium vapors are flowing inward with correspondingly decreasing velocities. At the highest level reached by iron and aluminum, the mean velocity is zero; at lower levels the direction of flow is reversed and the outward velocity increases with further decrease of elevation.<sup>1</sup>

The velocities represented in the third column refer to the undisturbed disk of sun, and appear to follow from spectrographic integration over wide areas of the surface.<sup>2</sup> A slit 0.1 inch long and 0.004 inch wide on the 18-inch image in the 150-foot telescope would, under perfect conditions, cover 1,000,000 square miles on the sun's surface. With unavoidable errors in guiding and with ordinary conditions of the seeing, the area is multiplied several fold. If convection currents are upward over the small bright granules<sup>3</sup> and downward over the larger dark interspaces, the resultant Doppler effect would be an asymmetry on the violet side. The decrease of upward velocity with elevation brings about a balanced state, which apparently occurs at the level of lines of intensities 4-5 on Rowland's scale. Above this level the asymmetry is to the red, and grows more marked at increasing elevation where the absorption by the cooler downward-drifting vapors becomes the more effective factor in producing the spectral lines.

The question may be approached from another angle. The rapid rise of chromospheric matter from narrowly localized areas in the form of prominences suggests the possibility of an occasional uprush

<sup>1</sup> *Mt. Wilson Contr.*, No. 69, p. 21; *Astrophysical Journal*, **37**, 342, 1913.

<sup>2</sup> *Monthly Notices, R.A.S.*, **84**, 93, 1923.

<sup>3</sup> Professor Langley estimated that the bright granules occupy about one-fifth of the sun's surface while they emit at least three quarters of the light. See Young, *The Sun* (1895), p. 103.

of vapors, their diffusion through the action of high-level currents, and their gradual descent, analogous to the volcanic action of Krakatoa and Katmai and the spreading of the dust through the upper reaches of the terrestrial atmosphere. Another factor to be taken into consideration is the vaporization of meteoric matter in the upper atmosphere of the sun. The increase of mass would disturb the equilibrium between radiation-pressure and gravitation, and, through the settling of the material, induce a downward drift with a gradually decreasing velocity.

In a previous paper<sup>1</sup> on the constancy of wave-length of solar lines, we remarked that at any given level the convection currents reach a comparatively steady state, the resultant velocity differing in magnitude from level to level, and, in widely different levels, in sign. Across a surface described at any level there is a flux of material, but the mass in equilibrium is limited, and, when the limit is exceeded by addition from below or above, downward drifts must take place. The equilibrium is not static but kinetic, with a constant interchange of the material, whether held in equilibrium under an appreciable pressure-gradient or under radiation pressure.

The fourth column is a résumé of the pressure data discussed in the body of this paper, and in the fifth are given the mean angular and linear velocities from  $0^\circ$ - $45^\circ$  for  $Ca^+$  (St. John), for  $H\alpha$ , normal calcium, and low-level lines of iron and lanthanum (Adams). It is evident that high velocity correlates with high level, and, consequently, there is an eastward slip or drift of the upper layers of the atmosphere, a permanent east wind, decreasing in velocity with approach to the photosphere. In view of the interpenetration of the gases of the different levels, it is difficult, however, to conceive an enginery capable of maintaining this drift without an impulse from outside the sun.

In the sixth column the relative heights of the factors in the first five columns are interpreted in kilometers by means of Mitchell's measures of flash spectra.<sup>2</sup> In the case of weak lines it may be remarked that the high temperature of the low-lying

<sup>1</sup> *Mt. Wilson Contr.*, No. 223, p. 11; *Astrophysical Journal*, 55, 46, 1922.

<sup>2</sup> *Astrophysical Journal*, 38, 407, 1913.

vapors weakens them as absorption lines, but the high temperature and the great number of emitting centers in the line of sight near the limb strengthen them as bright lines in the flash. This results in relatively strong photographic action and an overestimate of their height on eclipse plates. The heights assigned to the low-level lines in the chart need to be reduced by some as yet unknown amount.

Although the chart represents our present interpretation of a wide range of observations, it is not to be taken as definitive, but rather as suggestive of lines along which investigations may profitably be directed. The way in which the different factors fall into a scheme based upon levels arrests the attention, and the cumulative effect creates a strong impression of the essential truth of the representation.

MOUNT WILSON OBSERVATORY  
March 1924

## CONVECTION CURRENTS IN STELLAR ATMOSPHERES

BY CHARLES E. ST. JOHN AND WALTER S. ADAMS<sup>1</sup>

### ABSTRACT

*Interpretation of relative displacements in stellar spectra.*—A study of spectrograms of bright stars taken at Mount Wilson with high dispersion and an application of results from solar investigations indicate: (1) The pressures in the atmospheres of Sirius, Procyon, and Arcturus at the level at which the absorption lines are produced are low and comparable with that in the sun. (2) The relative displacement of arc and enhanced lines is largest for Sirius and smallest for Arcturus and may be correlated with temperature. It is nearly equal in amount to that between high-level and low-level lines in the same spectra. (3) The hypothesis that this displacement is due to differences in radial velocity, the lines produced at the higher levels showing as in the sun a prevailing downward motion of the gases, seems to be an adequate explanation. (4) The results lend support to Campbell's hypothesis that convection currents may account, at least in part, for the *K* term in *B*-type stars.

The investigation by St. John and Babcock on pressures in the solar atmosphere and those of Fowler and Milne, Russell, and others, based on ionization theory, make it highly probable that the pressures in stellar atmospheres are extremely low. It is of interest, accordingly, to examine from our present point of view the results obtained from a study of the spectra of a few of the brighter stars made at Mount Wilson a number of years ago, and to apply to them the methods used in the case of the sun.

In the years 1909 and 1910 spectrograms of Sirius, Procyon, and Arcturus were obtained by Adams and Babcock, using the 60-inch reflector in the coudé form and a fixed auto-collimating prismatic spectrograph of 5.5 m focal length. The linear scale of the photographs was about 1.4 Å per mm at  $\lambda 4300$  and 6.2 Å at  $\lambda 6500$ . The spectrograms were measured by Adams, Miss Lasby, and Miss Ware, and the principal result of the investigation, the relative displacement between enhanced and arc lines, was discussed by Adams in a publication issued in 1911.<sup>2</sup> The values found are given in Table I on page 44.

At the time the investigation was made it seemed reasonable to ascribe the relative displacement of the enhanced lines toward the

<sup>1</sup> *Contributions from the Mount Wilson Observatory*, No. 279.

<sup>2</sup> *Mt. Wilson Contr.*, No. 50; *Astrophysical Journal*, **33**, 64, 1911.

red to the greater pressure-coefficients of such lines, and an attempt was made to calculate the pressures in the stellar atmospheres from the numerical values of the shifts of the lines. The discussion was regarded as distinctly speculative, but it was suggested that it indicated what might prove to be a satisfactory method of attack on the problem. In recent years numerous investigations have placed much more adequate analytical methods at our disposal. The studies of the relative behavior of solar and arc lines, the recognition of pole-effect in the arc, the classification of iron lines into pressure and temperature groups by Gale and Adams, and the extension of the temperature classification by King with the electric furnace to other elements have led to the possibility of interpreting

TABLE I  
ENHANCED LINES MINUS ARC LINES

	ENHANCED No.	ARC No.	DISPLACEMENT	
			A	km
Sirius.....	149	194	+0.014	+0.90
Procyon.....	113	349	.009	.58
Arcturus.....	86	715	+0.001	+0.08

the behavior of solar and stellar lines in much more satisfactory ways. The explanation of the prominence of the enhanced lines in the solar chromosphere, first suggested by Gale and Adams<sup>2</sup> as due to low density at high levels, has been confirmed fully by the ionization theory of Saha, and has removed the principal difficulty in accounting for this characteristic of the solar spectrum. The importance of considerations of level and radial motions at different levels, and the use of spectral lines according to the classes into which they are divided by their behavior in the laboratory, are most essential features of any discussion of the conditions in solar and stellar atmospheres.

The order of magnitude of the pressures in the atmospheres of these stars can be determined directly by measuring the displacements of groups of lines for which the pressure shifts differ considerably in the laboratory, a method already employed by

<sup>2</sup> *Mt. Wilson Contr.*, No. 58; *Astrophysical Journal*, 35, 45, 1912.

St. John and Babcock in the case of the sun.<sup>1</sup> Table II shows the result of such a comparison, on the stellar spectrograms already described, of lines of classes *c* and *d* of large pressure shift, with lines of classes *a* and *b* of small pressure shift. Only lines which are not enhanced have been used. The comparison is limited to lines reduced by the same arc standards in the reference spectrum so that pole-effect should be very nearly the same for the groups compared. The influences of radial motion and gravitational field are minimized by comparing groups of lines of nearly the same average wave-length which originate, so far as we may judge from solar analogy, at approximately the same height in the stellar atmospheres. In the case of Arcturus, a few manganese lines which have large pressure displacements<sup>2</sup> are included with those of groups *c* and *d* for comparison with groups *a* and *b*.

TABLE II  
RELATIVE DISPLACEMENTS OF LINES WITH LARGE AND  
SMALL PRESSURE-COEFFICIENTS

	NO. PLATES	Lines		DISPLACEMENT	PRESSURE (ATMOSPHERES)
		No.	Class		
Sirius.....	7	70 87	<i>c, d</i> <i>a, b</i>	+0.001	+0.4 ± 1.1
Procyon.....	4	71 158	<i>c, d</i> <i>a, b</i>	-0.003	-0.8 ± 0.9
Arcturus.....	11	175 247	<i>c, d, Mn</i> <i>a, b</i>	-0.002	-0.5 ± 0.8
Sun.....					+0.13 ± 0.06

The principal inference to be drawn from these results is that the pressures in the atmospheres of these stars are low and of the same order as that in the sun. A similar investigation by Salet,<sup>3</sup> based on a much smaller number of lines, gave provisional values of about three atmospheres in the case of Arcturus and one or two atmospheres for Procyon.

<sup>1</sup> *Mt. Wilson Contr.*, No. 278; *Astrophysical Journal*, **60**, 32, 1924.

<sup>2</sup> Monk, *Astrophysical Journal*, **57**, 222, 1923.

<sup>3</sup> *Astrophysical Journal*, **53**, 327, 1921.

Investigations of the solar spectrum have shown that the displacements toward the red are larger for strong or high-level lines than for weak or low-level lines. This seems to be due to differences in radial velocity at different levels, the prevailing tendency being downward in the upper levels. The present material provides a means of testing whether similar conditions prevail in the atmospheres of some of the brighter stars, and whether the relative displacement of the enhanced lines toward the red may not be correlated with their high level in the stellar atmospheres rather than with greater pressure shift.

TABLE III  
RELATIVE DISPLACEMENTS OF ENHANCED AND ARC LINES IN STELLAR  
SPECTRA COMPARED WITH DISPLACEMENTS OF ARC LINES  
OF HIGH AND LOW LEVEL

	No. PLATES	No. OF LINES		ENHANCED <i>minus</i> ARC	No. OF LINES		HIGH <i>minus</i> LOW
		Enhanced	Arc		High- Level	Low- Level	
Sirius.....	4	141	176	+0.016 A	64	113	+0.015 A
Procyon.....	4	107	278	.008	73	222	.006
Arcturus.....	14	316	1070	.001	80	666	.002
Sun.....		4 (Ti)	13 (Ti)	+0.004	Mean Fe lines		+0.003

For this purpose two groups of arc lines have been formed, one consisting of lines which in the flash spectrum, according to Mitchell,<sup>1</sup> rise to heights of 600 km or more, and the other of lines whose heights are less than 600 km. These groups are designated as "high" and "low" level lines in Table III. The relative displacements of these groups of lines have been determined from the measurements of the stellar spectra, and the results are given in the last column of the table. The differences enhanced *minus* arc lines are given in the fifth column. For purposes of comparison there are added the relative displacements in the solar spectrum for high- and low-level iron lines in the same region and for a few enhanced and arc lines of titanium with heights in the flash spectrum of 1200 and 400 km, respectively.

<sup>1</sup> *Astrophysical Journal*, 38, 407, 1913.

The striking features of these results are: First, the close agreement of the displacements between enhanced and arc lines with those between high- and low-level arc lines; second, the decrease in the values from Sirius through Procyon to Arcturus. The evidence accordingly makes it highly probable that the relative displacement toward the red of the enhanced lines is due to downward motion of the gases at the high levels at which these lines are produced, and that this motion is greatest in the stars of high temperature and progressively smaller in those of lower temperature.

The values of the displacements toward the red of the stellar lines given in Table II are referred to the corresponding lines in the solar spectrum. The absolute values are given in Table IV together with the resulting velocities in kilometers. The spectral types are added for comparison, as well as the temperatures according to the results of Seares.<sup>1</sup>

TABLE IV  
RELATION OF DISPLACEMENT TO TEMPERATURE

	Type	Temperature	Displacement High-Level minus Low-Level	
Sirius.....	A2	8800°	+0.018 A	+1.20 km
Procyon.....	F3	6400	.010	0.67
Arcturus.....	K0	3900	.005	.34
Sun.....	G0	5800	+0.003	+0.20

It may fairly be assumed that convection currents are more pronounced the higher the temperature of the star, and the displacements given in Table III, interpreted as Doppler effects, are in agreement with this conclusion. The fact that Arcturus with a temperature lower than that of the sun shows slightly greater convection currents is not opposed to this view, since Arcturus is a giant star of immense size and extremely low density as compared with the sun, and the presence of more active radial motions is perhaps to be expected.

In this connection it is of interest to refer to some unpublished measurements upon the spectra of certain other stars made in recent years. These spectra were obtained with the three-prism

<sup>1</sup> *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, 55, 165, 1922.

Cassegrain spectrograph and a camera of 102-cm focal length. Although the linear scale is much smaller than that of the spectrograms of the bright stars discussed here, the results when based upon numerous lines are entitled to considerable confidence. Among the stars observed in this way were  $\alpha$  Orionis,  $\alpha$  Arietis,  $\beta$  Andromedae,  $\beta$  Geminorum,  $\alpha$  Tauri, and  $\delta$  Cephei at maximum and minimum of light. The measurements on the first five of these stars give results in very fair agreement with that obtained for Arcturus, the difference between enhanced and arc lines being  $+0.006 \text{ \AA}$ , or  $\pm 0.40 \text{ km}$  in terms of velocity. These are all giant stars of type Ko or later. The results for  $\delta$  Cephei are of especial interest. The differences between enhanced and arc lines are the largest found for any star, and differ at maximum and minimum of light.

	Enhanced <i>minus</i> Arc Lines	
Maximum.....	$+0.036 \text{ \AA}$	$+2.4 \text{ km/sec.}$
Minimum.....	$+0.014$	$+0.9$

The presence of strong convection currents in a variable star of this class is very probable, especially if the pulsation theory of the cause of variability is accepted. That these currents should be less active at minimum of light would be a reasonable conclusion and in agreement with the results of measurement.

The effects of anomalous dispersion and generalized relativity are both eliminated from these results, the former by considerations of the extremely low pressures involved, the latter by the fact that the observations are differential and deal with lines which occur in the same spectral region, and, so far as we may judge from solar conditions, differ only in level.

If this interpretation of the relative displacements of lines of high and low level is correct, and if we can assume that the downward drift of the gases at high levels in stellar atmospheres increases in velocity with increase of temperature, we may find in it an explanation of the K term in B-type stars. This hypothesis was suggested several years ago by Campbell,<sup>1</sup> and the present results favor it strongly. The principal lines used in the determination of the radial velocities of B-type stars are all high-level lines of ele-

<sup>1</sup> *Bulletins of the Lick Observatory*, 8, 71 (No. 257), 1914.

ments such as *H*, *He*, *Mg*, *Si*, *O*, and *N*. In the spectrum of Sirius the relative displacement of ordinary arc lines for moderate differences of level is 0.018 Å at  $\lambda 4500$ , or 1.2 km, while for *Hγ* it is about 2.0 km. It seems quite reasonable to conclude from these results that at the higher temperatures of the B-type stars the downward velocities of the convection currents will attain a value of the order of 4 km, the amount of the *K* term.

The presence of strong convection currents in the atmospheres of the stars has frequently been suggested as the explanation of the difference in velocity between the bright hydrogen lines and the absorption lines of the Me-type stars. The bright lines are displaced toward the violet with reference to the absorption lines by amounts varying from less than 0.1 Å to nearly 0.3 Å, or from about 5 to nearly 20 km in velocity.<sup>1</sup> The difference has been shown by Ludendorff and by Merrill to be directly correlated with the period of the variation of light of the stars, and Merrill has also found that the difference is in general larger the greater the range of variation in magnitude. The effect seems to be entirely capable of explanation, qualitatively at least, by the presence of ascending currents of hydrogen gas which produce bright lines shifted to the violet with reference to the slowly descending currents which produce the absorption lines. This is in agreement with St. John's results for the bright *H* and *K* lines in the sun, which show upward motion with reference to the absorption lines. It seems quite reasonable to conclude that in giant variable stars of the Me class similar currents of much greater intensity are present, and that the velocity of these currents is higher the greater the amount of the variation of the stars.

MOUNT WILSON OBSERVATORY  
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<sup>1</sup> Merrill, *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, 58, 215, 1923.

# THE SUN'S MOTION AND THE MEAN PARALLAXES OF STARS OF DIFFERENT APPARENT MAGNITUDES<sup>1</sup>

By FREDERICK H. SEARES

## ABSTRACT

*The sun's speed.*—The mean parallaxes of stars of different apparent magnitudes calculated from parallactic motions with a constant solar speed do not agree with those derived from the luminosity and density functions. Although the quadratic exponentials used to represent the distribution of luminosity and density are subject to serious limitations, the mean parallaxes computed with their aid appear to be reliable within a small percentage to the seventh apparent magnitude. The inference is that the solar speed varies with the brightness of the reference stars, and a comparison of the two series of results indicates the relation

$$V_0 = 12.9 + 1.21 m \quad (m = 1 \text{ to } 7), \quad (18)$$

or

$$V_0 = 19.4 + 2.8 M \quad (M = -1.9 \text{ to } +0.7). \quad (16)$$

The second formula is in close agreement with the results of Strömberg's investigation in *Contribution* No. 245, which gave  $V_0 = 18.8$  km/sec. for 800 giants, and  $V_0 = 31.7$  for 415 dwarfs. Equation (16) therefore appears to hold to at least  $M = +4.6$ , the mean absolute magnitude of Strömberg's group of dwarfs. The lower limit of  $m$  in (18) can in consequence be extended to the twelfth or thirteenth magnitude without risk of serious error.

*Mean parallaxes for stars of different apparent magnitudes.*—With the aid of (18) revised mean parallaxes have been calculated from parallactic motions to  $m = 13$  (Table IV). As compared with results found with a constant solar speed, the parallaxes of the brightest stars are increased by 25 or 30 per cent, while those of the eleventh to the thirteenth magnitudes are decreased by about the same amount.

Strömberg's discussion of stellar velocities in *Contribution* No. 245<sup>2</sup> brought to light the important fact that the sun's speed depends upon the intrinsic brightness of the reference stars. In the mean, 800 giants gave  $V_0 = 18.8$  km/sec., while for 415 dwarfs the result was 31.7. The mean absolute magnitudes of the groups thus compared were approximately 0 and +4.6, respectively.

An important consequence of the dependence of solar speed on the intrinsic brightness of the reference objects is a modification of the mean parallaxes calculated from parallactic motion for stars of different apparent magnitudes (all proper motions together).

<sup>1</sup> *Contributions from the Mount Wilson Observatory*, No. 281.

<sup>2</sup> *Astrophysical Journal*, 56, 265, 1922. See Table I.

These have hitherto been determined from the secular parallaxes for each value of  $m$  with a constant value of the solar speed,<sup>1</sup> corresponding closely with the value found by Strömberg for the giants. But the mean absolute magnitude of stars of apparent magnitude  $m$  increases about 0.4 mag. for every unit of increase in  $m$ . Thus the mean  $M$ 's for  $m=1$  and  $m=11$  differ by four magnitudes; and in view of Strömberg's result the use of the same solar speed for all values of  $m$  will in general give only an approximation for the mean parallax of stars of a given magnitude.

It has probably been noted by everyone who has dealt with the statistical problems of stellar distribution that mean parallaxes thus found do not agree with those calculated from the fundamental distribution functions for luminosity and density. The natural tendency has been to accept the results from parallactic motion and throw the discordance upon the luminosity and density functions. This, however, is to underestimate the reliability of these functions. It is true that the relative numbers of stars which are very faint intrinsically is still unknown, and that the values of the stellar density for distances greater than a few hundred parsecs are uncertain; but the distribution of the more luminous stars in the neighborhood of the sun is known with very good approximation—so good, in fact, that from the discrepancy in the mean parallaxes found by the two methods something can be learned as to the characteristics of the sun's motion, at least with respect to the more luminous stars. It is the purpose of this note to see how far the present data will permit us to go in this direction.

When the distribution functions for stellar luminosity and density are written in the form

$$\phi(M)dM = dM e^{p+qM+rM^2}, \quad (1)$$

$$\Delta(\rho) = e^{h+k(\log \rho)+l(\log \rho)^2}, \quad (2)$$

where

$$M = m - 5 \log \rho \quad (M, \text{ Kapteyn's scale; } \rho \text{ in parsecs}), \quad (3)$$

<sup>1</sup> Kapteyn adopted 19.5 km/sec.

the theoretical expression for the arithmetical mean parallax of the stars of magnitude  $m$  has the form<sup>1</sup>

$$\log \bar{\pi}_m = K'_1 + K_2 m, \quad (4)$$

in which

$$K'_1 = \delta'/2\gamma_1 \quad K_2 = -5r/\gamma_1, \quad (5)$$

$$\delta' = 2.5a' + k - 5q \quad \gamma_1 = l + 25r, \quad a' = 1/\text{Mod.} \quad (6)$$

It is shown in *Contribution* No. 273, that with<sup>2</sup>

$$q = +0.4278 \quad r = -0.07944 \quad (7)$$

equation (1) represents satisfactorily the distribution of the absolute magnitudes of stars brighter than  $M = +7$  (int. scale). Further, it is shown that (2), with appropriate values of  $k$  and  $l$ , represents the relative densities within the limits of  $\log \rho$  given in Table VII of that paper.

The data relating to the density function essential for the present inquiry are repeated in Table I below. The coefficients

TABLE I  
COEFFICIENTS OF DENSITY FUNCTION

Median $\log \rho$ ....	1.1	1.5	1.9	2.3	2.6
$k$ .....	+0.069	+0.327	+0.962	+2.42	+5.48
$l$ .....	-0.115	-0.235	-0.438	-0.806	-1.51
$\kappa_1$ .....	1.45	1.49	1.56	1.67	1.87
$r$ .....	$\pm 0.329$	$\pm 0.320$	$\pm 0.306$	$\pm 0.286$	$\pm 0.255$

$k$  and  $l$  are functions of the distance, and any pair of values is applicable throughout a range of about 0.8 in  $\log \rho$  on either side of the corresponding median  $\log \rho$  given in Table I. The densities

<sup>1</sup> The derivation of the formula for the mean parallax of stars of a given  $m$  and  $\mu$  is given in *Mt. Wilson Contr.*, No. 273. The formula for a given  $m$ , i.e., for all proper motions collectively, follows at once by omitting  $P(\tau)d\tau$ , the distribution function for tangential velocity, from (13) and (15). The coefficients  $\alpha$ ,  $\beta$ ,  $\gamma$ , in (17) are then to be replaced by  $\alpha_1$ ,  $\beta_1$ ,  $\gamma_1$ ; whence

$$\log \bar{\pi}_m = -\frac{\beta_1}{2\gamma_1}$$

where  $\beta_1$  and  $\gamma_1$  are defined by (28) and (22). By (20), the equation for  $\log \bar{\pi}_m$  is obtained by replacing  $3a' = 3/\text{Mod.}$  in the formula for  $\beta_1$  by  $2.5/\text{Mod.} = 5.756$ .

<sup>2</sup> Kapteyn and van Rhijn, *Mt. Wilson Contr.*, No. 229; *Astrophysical Journal*, 55, 242, 1922.

themselves are certainly of the right order of magnitude up to a thousand parsecs, and, within the intervals specified, the representation of these densities by (2) is usually within 1 per cent.

Equation (4) is a theoretical consequence of (1) and (2), and the limitations of the latter formulae naturally impose restrictions on equation (4) itself. Since the failure of (1) is serious for  $M > +8$ , it is clear that (4) cannot be applied to values of  $m$  so large as to include any considerable percentage of stars intrinsically fainter than this limit. Table II of *Contribution* No. 271<sup>1</sup> shows what is to be expected in this respect. Among the stars brighter than  $m=5$  there is (theoretically) no object intrinsically fainter than  $M=+8$ ; and even for  $m=10$  to 11 the number having  $M > +8$ , assuming (1) to hold rigorously for all values of  $M$ , is only one-sixth of 1 per cent of the total. With full allowance for the departure of (1) from the true distribution (see Fig. 2, *Contribution* No. 273) the percentage beyond the limit of applicability of (1) must still be very small. Little disturbance from this source is therefore to be feared for stars brighter than the eleventh or twelfth apparent magnitude.

There remains still the restriction imposed by the limited usefulness of the numerical values of  $k$  and  $l$ . Here the question is whether the stars of a given  $m$  are not so widely scattered in space that a single pair of values is insufficient to represent the densities within this range. Were this the case, (4), which presupposes a constant  $k$  and  $l$  for a given  $m$ , would of course be inapplicable.

From (1) and (2) it follows that the values of  $\log \pi_m$  have a Gaussian distribution about the mean  $\log \pi_m$ . The modulus  $\kappa_1$  of the distribution function for  $\log \pi_m$  is determined by<sup>2</sup>

$$\kappa_1^2 = -\gamma_1. \quad (8)$$

The values of  $\kappa_1$  and of  $r=0.477/\kappa_1$ , the probable dispersion in  $\log \pi_m$ , found from (6), (7), and the values of  $l$  in Table I, are given in the last two lines of that table. The dispersion in  $\log \pi_m$  varies appreciably with the distance. For the two extreme values,  $r=\pm 0.329$  and  $\pm 0.255$ , the distribution of the stars within

<sup>1</sup> *Astrophysical Journal*, 59, 17, 1924.

<sup>2</sup> See equation (17), *Mt. Wilson Contr.*, No. 273, noting that  $\gamma$  is here replaced by  $\gamma_1$ .

successive intervals of 0.2 in  $\log \pi_m$  is that shown in Table II. The zero point for the intervals in the first column is the value of  $\log \pi_m$ .

TABLE II  
DISTRIBUTION OF STARS ABOUT  $\log \pi_m$

INTERVAL IN $\log \pi_m$	FREQUENCIES	
	$r = \pm 0.329$	$r = \pm 0.255$
0.0 to 0.2 . . . . .	0.32	0.40
.2 to .4 . . . . .	.27	.31
.4 to .6 . . . . .	.19	.17
.6 to .8 . . . . .	.12	.08
0.8 to 1.0 . . . . .	.06	.03
> 1.0 . . . . .	0.04	0.01
Totals . . . . .	1.00	1.00

For the less favorable case, 90 per cent of the stars lie within a range of 0.8 in  $\log \pi_m$  on either side of the mean value. Ten per cent therefore fall outside the limits of distance within which the values of  $k$  and  $l$  have been regarded as constant. This, however, is not serious, because the range given for  $k$  and  $l$  is that within which the representation of the densities is almost exact. The interval can be extended to 1.0 or 1.2 on either side before the failure of (2) becomes fatal. In other words, the limitations of (2) do not appreciably affect the validity of equation (4). The only point to be noted is that the values of  $k$  and  $l$  used with (4) must in each case correspond rather closely to the particular median  $\log \rho$ ; otherwise, their range of applicability will not cover the interval throughout which the stars in question are distributed. This condition is easily satisfied by successive approximations.

This preliminary discussion shows that, in so far as theoretical defects are concerned, (4) should lead to values of  $\bar{\pi}_m$  which are reliable within a very small percentage for all magnitudes down to  $m = 10$  or 11. The question as to the effect of uncertainty in the numerical values of the coefficients now available is another matter. The results given in *Contribution* No. 273, together with those of Kapteyn and van Rhijn,<sup>1</sup> indicate that the values of  $q$  and  $r$  in (7), and the first three pairs of values of  $k$  and  $l$  in Table I are probably

<sup>1</sup> *Mt. Wilson Contr.*, No. 188; *Astrophysical Journal*, 52, 23, 1920.

near the truth. For the last two pairs of density coefficients the circumstances are much less favorable.

To make this clear, the methods used for calculating these coefficients must be recalled. For regions near the sun the densities, as in the case of those represented by the first four pairs of coefficients in Table I, are usually found in connection with the derivation of the luminosity function (1): The statistical data on proper motions and magnitudes give the number of stars,  $n$ , having the magnitude  $m$  and motion  $\mu$ . The mean distance of these stars can be found from the mean-parallax formula. Then, knowing the distribution law of  $\log \pi$  with respect to  $\log \pi$  and the dispersion, we can find how the  $n$  stars are distributed among a series of concentric shells having the sun at their center. By applying this procedure to all possible combinations of  $m$  and  $\mu$ , and summing the results for each shell, the total stellar content of each shell can be determined. The division of the total numbers of stars thus found by the volumes of the respective shells then gives the relative densities.

Practically, the matter is not so simple. Proper motions for stars fainter than the twelfth or thirteenth apparent magnitude are lacking, and the data for small proper motions of stars of all magnitudes are relatively uncertain. Hence we do not in any case obtain the total stellar content of any shell, and we are obliged to derive the densities by comparing the number of stars in each shell which fall within rather narrow limits of absolute magnitude. With increasing distance these limits constantly shift toward the brighter end of the scale of luminosity, so that the percentage of the total content of each shell actually dealt with, even when referred to unit volume, falls off rapidly, and, at the same time, becomes more and more unreliable because of increasing uncertainty in the statistical data on proper motions. The result is that the uncertainty in the densities also increases rapidly with the distance.

As for the data in Table I, the errors in  $k$  and  $l$  for median  $\log \rho = 1.1, 1.5$ , and  $1.9$  are probably unimportant for the present purpose. The values for median  $\log \rho = 2.3$ , which represent the densities up to about a thousand parsecs, must be of the right order of magnitude, but are probably too uncertain for use in any close comparison of calculated and observed values of  $\bar{\pi}_m$ . This conclusion is

based upon the characteristics and internal agreement of the data, and seems to be confirmed by the comparison given in Table III, presently to be described.

The last pair of values for  $k$  and  $l$  is by Kapteyn and van Rhijn, and was found by an entirely different method. It is well known that the numbers of stars in each unit interval of  $m$  can be well represented by the exponential function<sup>1</sup>

$$N(m) = e^{a+bm+cm^2}, \quad (9)$$

and that  $a$ ,  $b$ , and  $c$  are connected with the coefficients of (1) and (2) by the relations

$$l = \frac{25cr}{r-c}, \quad (10)$$

$$k = 5q - 6.908 + \frac{l+25r}{5r} (b-q). \quad (11)$$

With values of  $b$  and  $c$  derived from the star counts given in *Groningen Publication* No. 27, and  $q$  and  $r$  as given in (7), Kapteyn and van Rhijn used equations (10) and (11) to calculate  $k$  and  $l$  for the distant stars.

The coefficients thus found are applicable for values of  $\log \rho > 1.8$ , and, from the behavior of the other values of  $k$  and  $l$ , it is assumed that the corresponding quadratic function represents the densities to about  $\log \rho = 3.4$ . The median  $\log \rho$  is accordingly 2.6.

The scale to which the star counts of *Groningen Publication* No. 27 are referred is known, however, to be in error, with the result that  $b$  and  $c$  are still affected by considerable uncertainty. The dependence of  $\log \bar{\pi}_m$  upon these coefficients can be seen by combining equations (10) and (11) with (4) to (6), which eliminates  $k$  and  $l$ . The result is

$$K_1' = \frac{1}{10r} \left\{ -0.230 \left( \frac{r-c}{r} \right) + b - q \right\} \quad (12)$$

$$K_2 = -\frac{1}{5} + \frac{c}{5r}. \quad (13)$$

<sup>1</sup> It is probable that here, as in the case of the density function, the coefficients are themselves functions of  $m$ , and that any given pair of values of  $b$  and  $c$  (the coefficient  $a$  is irrelevant) has a limited range of applicability.

<sup>2</sup> Kapteyn and van Rhijn, *Mt. Wilson Contr.*, No. 188; *Astrophysical Journal*, 52, 23, 1920. Equations (17) and (19).

The errors of  $b$  and  $c$  are unknown, but that affecting  $c$  is probably large. Since  $K_2$  appears in equation (4) multiplied by  $m$ , and since the distant stars are apparently faint, it is not likely that this equation can be used successfully with the last pair of density coefficients given in Table II.

TABLE III  
CALCULATION OF THE SUN'S SPEED

$m$	$P$	$\log \bar{\pi}_m$ from $P$ $V_0=19.5$	$k$	$l$	$K'_1$	$K_2$	$\log \bar{\pi}_m$ from (4)	Diff.	$V_0$	$M$
1.....	0.247	-1.222	+0.05	-0.11	-0.874	-0.189	-1.063	+0.159	13.6	-1.90
2.....	.180	1.359	.15	.15	.881	.186	1.253	.106	15.3	1.47
3.....	.131	1.498	.27	.21	.884	.181	1.427	.071	16.6	1.04
4.....	.0959	1.633	.42	.275	.894	.176	1.598	.035	18.0	0.60
5.....	.0698	1.770	0.68	.36	.915	.169	1.760	+0.010	19.0	-0.17
6.....	.0508	1.910	1.03	.46	.949	.162	1.921	-0.011	20.1	+0.26
7.....	.0370	2.046	1.47	.57	0.994	.155	2.080	.034	21.1	0.69
8.....	0.0268	-2.186	+2.04	-0.72	-1.044	-0.1465	(-2.216)	-0.030	(20.9)	+1.12

The actual application of formula (4), which is very simple, gives the results in Table III. The second column contains values of the secular parallax  $P$  for the first eight magnitudes, taken from Table 25 of *Groningen Publication No. 29*. The values of  $\log \bar{\pi}_m$  in the third column are those corresponding to a constant solar speed of 19.5 km/sec., computed by the usual formula

$$\bar{\pi}_m = 4.74 \frac{P}{V_0} \quad (14)$$

These were used to obtain values of  $k$  and  $l$ , which were interpolated from Table II with  $\log \rho = -\log \bar{\pi}_m$  as argument. The sixth, seventh, and eighth columns contain the values of  $K'_1$ ,  $K_2$ , and  $\log \bar{\pi}_m$  from equations (4) to (6). As a matter of fact, the tabular data for  $k$ ,  $l$ ,  $K'_1$ , etc., are the results of a second approximation, the first having been based on the values of  $\log \bar{\pi}_m$  given by (14).

A comparison of the third and eighth columns (the differences are in the ninth column) shows that the mean parallaxes of the brightest stars calculated with  $V_0=19.5$  km/sec. are about two-thirds those found from the distribution functions for density and luminosity. The difference gradually decreases and changes sign between the fifth and sixth magnitudes. The comparison cannot safely be

extended beyond  $m = 7$  because the values of  $k$  and  $l$  then fall within the region of uncertainty affecting the data of Table II. For the first six or seven magnitudes, however, the revised mean parallaxes in the eighth column must be nearer the truth than those based on a constant solar speed.

As already suggested, these results can be used in conjunction with the secular parallaxes to calculate  $V_0$  itself. Equation (14) gives directly the values in the last column but one of Table III. These correspond to the mean absolute magnitudes given in the last column, derived from the formula<sup>1</sup>

$$\bar{M} = -2.33 + 0.4315 m. \quad (15)$$

The values of the solar speed are well represented by the linear formula

$$V_0 = 19.4 + 2.8 \bar{M}. \quad (16)$$

If it can be supposed that the values of  $V_0$  found by Strömberg for giants and dwarfs<sup>2</sup> represent two points of a linear relation, the corresponding formula would be

$$V_0 = 18.8 + 2.8 \bar{M}. \quad (17)$$

Although the agreement of these two expressions is perhaps accidentally close, their accordance gives considerable weight to the results for the sun's motion referred to stars of different intrinsic brightness.

The combination of (16) and (15) gives  $V_0$  as a function of apparent magnitude:

$$V_0 = 12.9 + 1.21 m. \quad (18)$$

This formula can now be used with (14) to calculate the mean parallaxes for the fainter apparent magnitudes. The secular parallaxes are known to  $m = 13$ , and since the mean absolute magnitude for this limit is about  $+3.3$ , no extrapolation of (17) is implied,

<sup>1</sup> *Mt. Wilson Contr.*, No. 271; *Astrophysical Journal*, **59**, 11, 1924. See equation (14).

<sup>2</sup> See opening paragraph.

and the parallaxes thus found should accordingly be a good approximation. The second column of Table IV gives results based on the linear formula (18) for all values of  $m$ . The deviations from the values in the third column, which correspond to the eighth column of Table III, are unimportant. Within the interval considered, the linear expression for  $V_0$  is therefore satisfactory.

TABLE IV  
ARITHMETICAL MEAN PARALLAXES OF STARS OF  
DIFFERENT APPARENT MAGNITUDES

$m$	Adopted, $V_0$ by Eq. (18)	From Distr. Functions	$V_0$ const. 19.5 km/sec.
1.....	0".0830	0".0865	0".0600
2.....	.0558	.0558	.0437
3.....	.0376	.0374	.0318
4.....	.0257	.0252	.0233
5.....	.0175	.0174	.0170
6.....	.0120	.0123	.0123
7.....	.00820	0.00832	.00899
8.....	.00562	.....	.00651
9.....	.00387	.....	.00471
10.....	.00266	.....	.00340
11.....	.00184	.....	.00248
12.....	.00126	.....	.00177
13.....	0.00088	.....	0.00129

The last two or three values of  $\bar{\pi}_m$  are perhaps slightly affected by the fact that the interval within which (15) holds has been somewhat exceeded. This equation presupposes the validity of (1) as an expression for the luminosity function; but (1) fails for  $M > 8.0$ . But since the percentage of the stars of the thirteenth apparent magnitude having  $M > 8.0$  is small,<sup>1</sup> the value of  $\bar{M}$  calculated by (15) cannot be seriously in error.

The very considerable change produced in the mean parallaxes by taking into account the dependence of solar motion upon the brightness of the reference objects may be seen by comparing the adopted results with those in the last column of Table IV, which are based on the constant value  $V_0 = 19.5$  km/sec.

MOUNT WILSON OBSERVATORY  
April 1924

<sup>1</sup> See p. 53.

# STANDARD WAVE-LENGTHS AND REGULARITIES IN THE SPECTRUM OF THE IRON ARC

By W. F. MEGGERS

## ABSTRACT

*Regularities in the spectrum of the iron arc.*—The wave-lengths of selected lines in the arc spectrum of iron form a system of secondary and tertiary standards whose accuracy in relative value can now be tested with the aid of the recent discovery of regularities in this spectrum. From the vacuum wave-numbers which represent standard wave-lengths in a selected group of multiplets, the relative values of 27 sets of spectral terms are calculated to 8 significant figures. Among these terms occur combinations giving rise to 84 multiplets belonging to the triplet, quintet, and septet systems. About 800 spectral lines, ranging in wave-length from 2276 Å to 9768 Å, and including most of the internationally adopted standards, are thus represented with considerable precision. Critical tests of the accuracy in relative values of such standards are made (1) from the recurrence of differences of wave-number which should be exactly the same in all multiplets involving one set of terms in common, and (2) from cyclical relations of multiplets in groups of four such that the wave-numbers for any one of them may be calculated from those of the other three. It appears that most of the wave-lengths of lines in any one multiplet are correct in relative value to one part in 2,000,000 or 3,000,000 except in the ultra-violet where errors as large as one part in 400,000 exist. The lines which are easily shifted by pressure and exhibit "pole-effect" are seen to originate in electron transitions from the outermost orbits of the excited iron atom. These effects can be eliminated and the accuracy of standard wave-lengths increased by the adoption of the *iron arc in vacuo* as the source of secondary and tertiary standards.

## I. INTRODUCTION

Many years of effort to establish a system of wave-length standards culminated finally in the adoption of the so-called "international system," with which all spectroscopists, astronomers, and metrologists are now familiar. For most of the important phases in the development of this subject the reader is referred to papers in which the history of standard wave-lengths has been reviewed.<sup>1</sup>

The purpose of the present discussion is to point to the most recent developments, to give a new presentation of the data, and to draw some conclusions therefrom. Additional measurements of wave-length standards and the discovery of regularities in the spectrum of the iron arc are referred to. The data for several hundred spectral lines, including most of the secondary and tertiary standards, are presented in the form of term-values (energy-levels) and an

<sup>1</sup> Meggers, "Reports on Standard Wave-Lengths," *Journal of the Optical Society of America*, **5**, 308, 1921; **6**, 135, 1922. Article on the "Measurement of Wave-Lengths," *Glazebrook's Dictionary of Applied Physics*, **4**, 882, 1923; Kayser and Konen, *Handbuch der Spectroscopie*, **7**, 405, 1924.

energy diagram showing the term combinations involved in the production of the lines; then certain properties of polyfold term combinations (constant wave-number differences, cyclical combination, relation of term combinations to pressure-shift and pole-effect) of special interest from the viewpoint of wave-length standards are discussed.

## II. THE WAVE-LENGTH DATA FOR THE IRON-ARC SPECTRUM

Since 1907, when the International Union for Co-operation in Solar Research recommended the determination of a new system of standards, the so-called "International Angstrom" (I.A.) scale has been developed through the adoption of a fundamental standard, about eighty secondary standards, and several hundred tertiary standards. The fundamental standard is the wave-length of the red radiation (6438.4696 Å) from cadmium; the secondary standards, each measured in terms of the fundamental standard by the etalon interferometer method of Fabry and Perot, consist of selected lines in the spectrum of the iron arc (3370.789-6750.163 Å); and the tertiary standards, from the same source, are determined relative to the secondaries by interpolation either with gratings or interferometers. This system of standards, comprising about four hundred values, was adopted in 1922 by the International Astronomical Union.<sup>1</sup> The present discussion is based principally upon these values, although use is made of additional values both within and outside the range of the adopted standards. In the visible and ultra-violet regions use is thus made of mean values of the determinations listed in the chapter on iron, in the seventh volume of *Handbuch der Spectroscopic*, by Kayser and Konen.

Lines which belong to pressure groups *c* and *d* (see IV. Discussion) having been found to exhibit "pole-effect" led to a change in the specifications of the international iron arc, which gives lower values for these wave-lengths. For these lines the lower values are used in this paper.

In the red and infra-red the recent "Interferometer Measurements of the Longer Waves in the Iron Arc Spectrum" are employed.<sup>2</sup>

<sup>1</sup> *Transactions of International Astronomical Union*, **1**, 35, 1922.

<sup>2</sup> Meggers and Kiess, *Sci. Papers, U.S. Bureau of Standards*, **19**, 273, 1924.

These values are about 0.005 Å lower than the adopted standards in the neighborhood of the fundamental standard, and in order to make them comparable to the adopted scale they have been increased by  $0.005 \frac{\lambda}{6438}$  Å. Unfortunately no wave-length measurements have ever been made in this region of the spectrum with the long arc so that it is not possible from observational data to identify the lines which show "pole-effect." Reference is also made in this paper to a "Redetermination of Secondary Standards of Wave Length from the New International Iron Arc,"<sup>1</sup> which deviates slightly from the adopted scale and thus raises the question of the ultimate precision which it is possible and desirable to obtain in a system of standards.

All of the wave-length data mentioned previously are reduced to values *in vacuo* by the aid of the Bureau of Standards tables,<sup>2</sup> and a wave-number (number of waves per cm *in vacuo*) is calculated for each line. From these wave-numbers the spectral terms in Table I are derived.

### III. TERMS FOR THE IRON-ARC SPECTRUM

When the series regularities of a spectrum are known it becomes possible to represent the wave-number corresponding to a particular line by the difference of two spectral terms, and all or most of the observed lines in such a spectrum can be represented by combinations of a limited number of series terms. Such a procedure has become possible in the case of the arc spectrum of iron through the recent discovery at the Bureau of Standards of regularities<sup>3</sup> in this complex spectrum. The spectral terms as known at present are listed in Table I, and the combinations of these which give rise to spectral lines are shown diagrammatically in Figure 1. On the diagram the terms are arranged horizontally with respect to spectral type and system and vertically in order of decreasing magnitude. The superscripts 3, 5, or 7 to the letters S, P, D, F, G, distinguish these types of terms in the triplet, quintet, and septet systems,

<sup>1</sup> Meggers, Kiess, and Burns, *Sci. Papers, U.S. Bureau of Standards*, **19**, 263, 1924.

<sup>2</sup> Meggers and Peters, *Bulletin, U.S. Bureau of Standards*, **14**, 697, 1918; also *Astrophysical Journal*, **50**, 56, 1919.

<sup>3</sup> Walters, *J.O.S.A. & R.S.I.*, **8**, 245, 1924.

respectively. Since it is possible that more than one sequence of terms of any one type exists, there is some uncertainty about assigning total quantum numbers to the terms, and they are therefore labeled with the same letters as in Walters' paper, except that K' is included with J, and V, W, and X have been added for the septet system, T' and Y to the quintet, and Z to the triplet systems. This makes it easy to identify on the diagram all of the

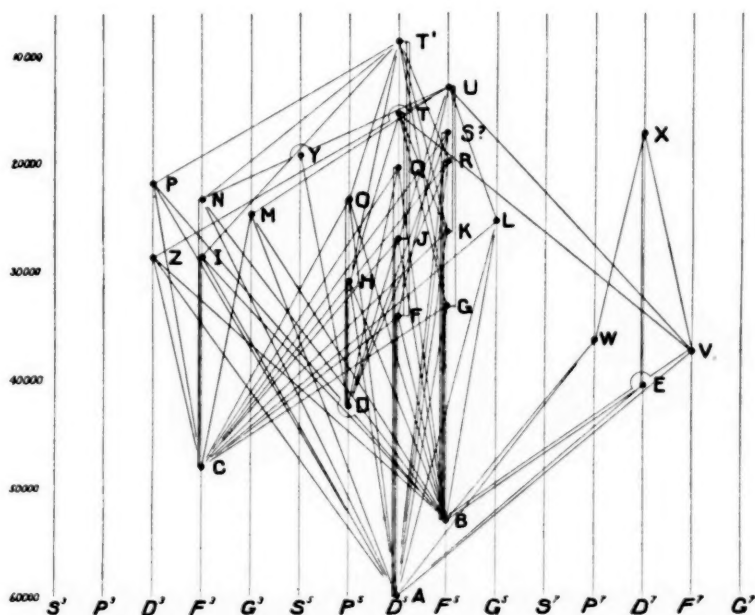


FIG. 1.—Diagram of terms (energy-levels), systems, and combinations for the spectrum of the iron arc.

multiplet structures published by Walters. Most of the terms are in reality composite; the different terms range in multiplicity from one to seven, but are necessarily represented on a small scale by a dot for each group. Reference must be made to Table I for the individual values for each term.

The A level, which is a fivefold D-term ( $D^5$ ) of the quintet system, is the lowest thus far known for the neutral iron atom, and is believed to represent the highest term in the arc spectrum of iron.

TABLE I  
SPECTRAL TERMS IN THE IRON-ARC SPECTRUM

Symbol	Term	<i>j</i>	Type	Combinations
A.....	60000.000	4	D <sup>5</sup>	E, V, W, F, G, H, I, Z, J, K, L, M, N, O, P, Q, R, S
	59584.064	3		
	59295.902	2		
	59111.860	1		
	59021.914	0		
B.....	53071.715	5	F <sup>5</sup>	E, V, W, F, G, H, I, Z, J, K, L, M, N, O, P, Q, R, S
	52623.220	4		
	52271.922	3		
	52014.198	2		
	51845.264	1		
C.....	48023.730	4	F <sup>3</sup>	F, G, I, Z, J, K, L, M, N, O, P, Q, R, S
	47439.934	3		
	47031.412	2		
D.....	42440.778	3	P <sup>5</sup>	H, J, O, Q, R, Y, S
	42272.974	2		
	42072.581	1		
E.....	40649.118	5	D <sup>7</sup>	A, B, X
	40437.552	4		
	40242.967	3		
	40087.488	2		
	39980.361	1		
V.....	37349.591	6	F <sup>7</sup>	A, B, T, X, U
	37154.131	5		
	37003.325	4		
	36889.057	3		
	36807.497	2		
	36755.163	1		
	36729.621	0		
W.....	36288.543	4	P <sup>7</sup>	A, B, X
	35819.136	3		
	35493.083	2		
F.....	34100.000	4	D <sup>5</sup>	A, B, C, T, U, T'
	33859.807	3		
	33660.291	2		
	33520.608	1		
	33449.500	0		
G.....	33125.444	5	F <sup>5</sup>	A, B, C, T, U, T'
	32833.167	4		
	32605.298	3		
	32440.400	2		
	32333.636	1		
H.....	30943.666	3	P <sup>5</sup>	A, B, D, T, T'
	30530.970	2		
	30267.254	1		

TABLE I—Continued

Symbol	Term	<i>j</i>	Type	Combinations
I.....	28692.725	4	F <sup>3</sup>	A, B, C, T'
	28194.899	3		
	27865.982	2		
Z.....	28677.358	3	D <sup>3</sup>	A, B, C, U
	28313.616	2		
	28062.638	1		
J.....	26904.046	4	D <sup>5</sup>	A, B, C, D, U, T'
	26492.862	3		
	26198.408	2		
	25982.872	1		
	25878.381	0		
K.....	26304.587	5	F <sup>5</sup>	A, B, C, T, U, T'
	25960.459	4		
	25671.229	3		
	25452.762	2		
	25307.826	1		
L.....	25156.012	6	G <sup>5</sup>	A, B, C, U
	25217.549	5		
	24742.652	4		
	24388.349	3		
	24143.575	2		
M.....	24620.766	5	G <sup>3</sup>	A, B, C, U
	24232.410	4		
	23920.005	3		
N.....	23313.802	4	F <sup>3</sup>	A, B, C, U, T'
	22837.235	3		
	22478.816	2		
O.....	23232.998	3	P <sup>5</sup>	A, B, C, D, T'
	22842.404	2		
	22590.420	1		
P.....	21824.617	3	D <sup>3</sup>	A, B, C, T'
	21321.925	2		
	21004.230	1		
Q.....	20374.172	4	D <sup>5</sup>	A, B, C, D
	20030.122	3		
	19768.643	2		
	19595.460	1		
	19508.691	0		
R.....	19742.710	5	F <sup>5</sup>	A, B, C, D, T'
	19495.617	4		
	19157.855	3		
	18981.972	2		
	18869.352	1		
Y.....	19104.981	2	S <sup>5</sup>	D

TABLE I—Continued

Symbol	Term	<i>j</i>	Type	Combinations
X.....	17184.166	5	D <sup>7</sup>	E, V, W
	16836.694	4		
	16565.386	3		
	16366.485	2		
	16236.038	1		
S.....	17088.14	5	F <sup>8</sup> ?	A, B, C, D
	16500.455	4		
	16077.283	3		
	15816.330	2		
	15714.529	1		
T.....	15323.000	4	D <sup>5</sup>	V, F, G, H, K
	14938.674	3		
	14606.127	2		
	14490.857	1		
	14404.924	0		
U.....	12994.502	5	F <sup>5</sup>	V, F, G, Z, J, K, L, M, N
	12622.032	4		
	12244.476	3		
	11963.346	2		
	11778.685	1		
T'.....	8649.502	4	D <sup>5</sup>	F, G, H, I, J, K, N, O, P, R
	8229.432	3		
	7950.192	2		
	7785.635	1		
	7742.626	0		

The azimuthal quantum number for this group is 3. The inner quantum numbers (*j*) for the sublevels are 0, 1, 2, 3, and 4. The lowest level in this group is 3D<sub>4</sub><sup>5</sup>, which corresponds to the normal state of the iron atom and indicates in terms of Bohr's ideas of atomic structure that the twenty-sixth electron of the iron atom is bound in a 3<sub>3</sub> orbit. The same conclusion has been arrived at by Gieseler and Grotrian<sup>1</sup> from observations of absorption in iron vapor.

It does not seem improbable that Q may be identified as 4D<sup>5</sup>, in which case a limit very close to 60,000 is obtained for such a sequence if it be assumed that the Rydberg law is obeyed. This corresponds to an ionization potential of 7.4 volts for iron, which is no doubt reasonably close to the truth. In Table I, therefore, 60,000.000 is retained as the absolute value of the highest term or lowest energy-level involved in the production of the iron-arc spectrum.

<sup>1</sup> *Zeitschr. Physik*, **22**, 245, 1924.

The relative values of terms listed in Table I are based upon a selected group of multiplets which represent the best wave-length determinations. So far as possible, only the multiplets which are composed entirely or chiefly of international adopted standards were used. The actual procedure was as follows: Since the B level plays so large a part in producing these standards, the separations of its sublevels were determined from its combinations with F, G, J, K, and L (see Table III) as 448.495, 351.298, 257.724, and 168.934. Assuming a value of 53071.715 for  $B_5$  gives 52623.221 for  $B_4$ , 52271.924 for  $B_3$ , 52014.200 for  $B_2$ , and 51845.265 for  $B_1$ . The values for F, G, J, K, L, M, and R terms are derived from their combinations with this B level. The multiplets AF and AG are well established so that the terms for A are accurately derived from those of F and G. From its combination with J the values for D result, and its combinations with H, N, O, Q, and S give values for the latter. Similarly, C is derived from K, L, and M and it in turn produces the values for I, P, and Z. The values for T are relative to F and G; U and T' are referred to F and K, respectively. Intersystem combinations AE and AV connect the septet with the quintet system, following which the X-terms result from combinations both with E and with V, and then W from its combination with X.

It will be recalled that according to the selection principle, if  $j$  and  $j'$  are the inner quantum numbers for individual levels in different groups, then spectral lines result from the combination of these levels for each combination of  $j=n$  with  $j'=n+1$ ,  $n$  or  $n-1$ , except that the combination  $j=0$  and  $j'=0$  is prohibited. This rule applied to the term-values in Table I, according to their combinations as shown in Figure 1, gives rise to 84 multiplets involving wave-lengths which range from 2276 Å to 9768 Å. The total number of lines thus expected is about 800, of which more than 95 per cent are recorded in tables of the iron-arc spectrum, while those not observed are, almost without exception, expected to be relatively faint on the basis of the intensity rule which accompanies the selection principles for azimuthal and inner quantum numbers. The use of these terms may now be illustrated by calculating the wave-lengths from a very few of the many combinations which are possible. The results are given in Table II.

A large majority of the international standards can be derived from the term-values in Table I with an error of 0.001 Å or less.

TABLE II  
WAVE-LENGTHS CALCULATED FROM SPECTRAL TERMS

Combination	$\lambda$ Vac.	$\lambda$ Vac.	$\lambda$ Air	$\lambda$ Standard
A <sub>1</sub> -F <sub>1</sub> .....	25591.254	3907.5850	3906.4825	3906.483
B <sub>4</sub> -G <sub>3</sub> .....	20017.920	4995.5240	4994.1350	4994.135
C <sub>2</sub> -I <sub>2</sub> .....	19165.430	5217.7280	5216.2704	5216.280
D <sub>3</sub> -J <sub>4</sub> .....	15545.732	6432.6337	6430.8600	6430.859
D <sub>2</sub> -H <sub>3</sub> .....	11329.309	8826.6637	8824.2435	8824.245

#### IV. DISCUSSION

The discovery of multiplets in the arc spectrum of iron is of considerable importance in connection with the establishment of a system of standard wave-lengths, because it permits tests of the accuracy in relative value of such standards. These tests are of two kinds: (1) The recurrence of wave-number differences which should be constant in every case where separations of the same sublevels are involved; and (2) cyclical relations of multiplets in groups of four such that the wave-numbers for any one of them may be calculated from those of the other three.

TABLE III  
CONSTANT WAVE-NUMBER DIFFERENCES

Multiplet	Separations of B Sublevels			
BF.....	448.507	351.292	257.725	168.929
		292	724	933
BG.....	505	302	729	925
		493	734	937
BJ.....	481	320	731	926
		298	733	941
BK.....	489	294	726	940
		506	716	930
BL.....	483	291	722	943
		296	702	
Mean.....	448.495	351.298	257.724	168.934

1. As examples of the first test, Table III gives the wave-number differences observed for the B level in multiplets BF, BG, BJ, BK, and BL.

The largest deviation from the mean of any column corresponds to an error of less than one part in 1,000,000 of the wave-length, and it appears that the values for most of the lines within any particular multiplet are correct to 0.001 Å in relative value.

Another example may be cited by giving the multiplet arrangement of the DH combination of levels, using the wave-lengths measured at the Bureau of Standards.<sup>1</sup>

	H <sub>3</sub>		H <sub>2</sub>		H <sub>1</sub>
D <sub>3</sub>	8688.6407		8387.7869		
	11506.124	412.704	11918.828		
	176.808		176.811		
D <sub>2</sub>	8824.2375		8514.0884		8327.0691
	11329.316	412.701	11742.017	263.715	12005.732
			200.392		200.392
D <sub>1</sub>			8661.9151		8468.4222
			11541.625	263.715	11805.340

The multiplet BF is selected as a third example, the wave-length values being taken from the recent redeterminations of the secondary standards by the Bureau of Standards<sup>2</sup> except three lines, 5269.4390, 5328.0424, and 5429.6998, which represent the Mount Wilson values<sup>3</sup> corrected to the Bureau of Standards' scale.

	F <sub>4</sub> 240.184	F <sub>3</sub> 199.518	F <sub>2</sub> 139.684	F <sub>1</sub> 71.106	F <sub>0</sub>
B <sub>5</sub>	5269.5390				
	18971.727				
	448.499				
B <sub>4</sub>	5397.1307	5328.0424			
	18523.228	18763.414			
	351.297				
B <sub>3</sub>	5501.4690	5429.6998	5371.4931		
	18171.928	18412.120	18611.636		
	257.724				
B <sub>2</sub>		5506.7826	5446.9194	5405.7781	
		18154.394	18353.914	18493.598	
	168.932				
B <sub>1</sub>			5497.5202	5455.6133	5434.5272
			18184.982	18324.666	18395.772

<sup>1</sup> Meggers and Kiess, *loc. cit.*

<sup>2</sup> Meggers, Kiess, and Burns, *loc. cit.*

<sup>3</sup> St. John and Babcock, *Astrophysical Journal*, **53**, 41, 1921.

The recurrence of wave-number differences with almost perfect constancy attests the accuracy in relative value of the wave-length measurements, and indicates that it may perhaps be desirable to retain the fourth decimal place whenever the probable errors of measurement are less than 0.001 Å. From the point of view of spectrum analysis it is readily seen that precision wave-numbers contain within themselves very strong evidence for the multiplet structures thus found in complex spectra.

2. The range of wave-lengths in any particular multiplet is comparatively small, but an important test of the accuracy in relative value over a much larger spectral range is furnished by groups of four multiplets in a cycle such as the following:

Cycle No.	Multiplets
1.....	BF, FT, TG, GB
2.....	DJ, JA, AH, HD
3.....	KU, UJ, JC, CK
4.....	FT, TG, GA, AF
5.....	DQ, QA, AH, HD
6.....	AR, RB, BQ, QA
7.....	AJ, JB, BR, RA
8.....	AF, FU, UL, LA
9.....	AG, GB, BQ, QA
10.....	BQ, QD, DJ, JB
11.....	DJ, JB, BS, SD
12.....	BJ, JA, AS, SB

We find for example, from Cycle 1:

Calc.	Obs.	Table I
$(B_1 - F_2) + (F_2 - T_1) - (B_1 - G_2) = (G_2 - T_1)$		
$18184.979 + 19169.445 - 19404.864 = 17949.560$	.554	.543
$(B_2 - F_3) + (F_3 - T_2) - (B_2 - G_3) = (G_3 - T_2)$		
$18154.392 + 19193.677 - 19408.808 = 17939.171$	.173	.171
$(B_3 - F_4) + (F_4 - T_3) - (B_3 - G_4) = (G_4 - T_3)$		
$18171.926 + 19161.328 - 19438.753 = 17894.501$	.491	.493
$(B_2 - F_2) + (F_2 - T_2) - (B_2 - G_2) = (G_2 - T_2)$		
$18353.908 + 18994.157 - 19573.789 = 17774.292$	.280	.273
$(B_3 - F_3) + (F_3 - T_3) - (B_3 - G_3) = (G_3 - T_3)$		
$18412.117 + 18921.139 - 19666.627 = 17666.629$	.621	.624

## Cycle 5:

Calc.	Obs.	Table I
$(D_3 - Q_3) + (A_3 - H_3) - (D_3 - H_3) = (A_3 - Q_3)$		
$22419.659 + 28640.403 - 11506.113 = 39553.949$	.832	.942
$(D_2 - Q_2) + (A_2 - H_2) - (D_2 - H_2) = (A_2 - Q_2)$		
$22504.332 + 28765.001 - 11741.994 = 39527.339$	.254	.349
$(D_1 - Q_1) + (A_1 - H_1) - (D_1 - H_1) = (A_1 - Q_1)$		
$24477.123 + 28844.603 - 11805.326 = 39516.386$	.367	.400
$(D_1 - Q_2) + (A_1 - H_2) - (D_1 - H_2) = (A_1 - Q_2)$		
$22303.934 + 28580.893 - 11541.616 = 39343.211$	.127	.217
$(D_3 - Q_2) + (A_3 - H_2) - (D_3 - H_2) = (A_3 - Q_2)$		
$22681.137 + 29053.104 - 11918.813 = 39615.428$	.336	.421

## Cycle 7:

Calc.	Obs.	Table I
$(A_4 - J_4) - (B_4 - J_4) + (B_4 - R_4) = (A_4 - R_4)$		
$33095.903 - 25719.186 + 33217.614 = 40594.331$	.366	.383
$(A_3 - J_3) - (B_3 - J_3) + (B_3 - R_3) = (A_3 - R_3)$		
$33091.172 - 25779.062 + 33114.052 = 40426.162$	5.976	6.209
$(A_2 - J_2) - (B_2 - J_2) + (B_2 - R_2) = (A_2 - R_2)$		
$33097.524 - 25815.785 + 33032.212 = 40313.951$	.949	4.020
$(A_1 - J_1) - (B_1 - J_1) + (B_1 - R_1) = (A_1 - R_1)$		
$33128.928 - 25862.388 + 32975.853 = 40242.393$	.452	.508
$(A_2 - J_1) - (B_2 - J_1) + (B_2 - R_1) = (A_2 - R_1)$		
$33313.057 - 26031.329 + 33144.904 = 40426.632$	.549	.640
$(A_3 - J_2) - (B_3 - J_2) + (B_3 - R_2) = (A_3 - R_2)$		
$33385.559 - 26073.518 + 33289.967 = 40602.008$	1.948	2.092
$(A_4 - J_3) - (B_4 - J_3) + (B_4 - R_3) = (A_4 - R_3)$		
$33507.135 - 26130.360 + 33465.410 = 40842.185$	.065	.145

It is seen that the wave-numbers of five lines in the GT multiplet computed from Cycle 1 are in the mean only 0.007 wave-number larger than the observed values, while the values computed from Table I are 0.004 smaller. These deviations correspond to relative errors of the order of 0.001 Å in the wave-length if all the error is assumed to occur in one line while the other three are regarded as perfect. The range of wave-lengths involved in Cycle 1, however, is rather small, but Cycles 5 and 7 test the scale throughout a much larger range. The mean difference, observed minus calculated, for five lines in the AQ multiplet is -0.080 wave-number from Cycle 5 and -0.083 from Table I, which may be interpreted as

follows: The observed wave-lengths in the region of 3100 Å are on the average 0.005 Å too large relative to the values longer than 3400 Å. A similar conclusion may be drawn from the deviation of observed and computed values for AR lines in Cycle 7, and in addition it appears that the best relative values near 2400 Å contain errors as large as one part in 400,000. It is recognized that the error in scale may be explained as resulting from a wrong value of the dispersion of air. This, however, would require in the refractivity for the ultra-violet an error of nearly 1 per cent, which is thought to be improbable.

That the calculation of one and the same multiplet from two or more cycles, involving successively shorter wave-lengths, provides a perfect standard scaling interval was called to my attention by Dr. Keivin Burns. Cycles 11 and 12 may be used for illustration. These contain BS (or SB) and BJ (or JB) in common; the multiplets and corresponding wave-length intervals may be arranged as follows:

DJ.....	6137-6430 Å
DS.....	3753-3852 Å
BJ.....	3820-3940 Å
BS.....	2733-2813 Å
AJ.....	2983-3059 Å
AS.....	2275-2329 Å

Here we have three pairs of multiplets in adjacent or slightly overlapping spectral regions, and each of the pairs should give exactly the same values for J-S. The values actually obtained for  $J_2-S_2$  are shown in Table IV.

TABLE IV  
VALUES OF  $J_2-S_2$

Combinations	Differences			Mean $J_2-S_2$
$D_{1,2,3}S_2-D_{1,2,3}J_2$ .....	10382.078	.077	.083	10382.079
$B_{1,2,3}S_2-B_{1,2,3}J_2$ .....	10382.036	.000	.013	10382.016
$A_{1,2,3}S_2-A_{1,2,3}J_2$ .....	10382.58	.08	.12	10382.26

This is a shift number or constant difference of rather large magnitude, which with three applications covers a wave-number interval of nearly 28,000. The multiplets DS and BJ overlap and are both contained in an interval of 187 Å; BS and AJ are adjacent

and cover only 326 Å; hence the observed wave-lengths for these groups may be regarded as being fairly accurate in relative value, and we may therefore lay the J-S "measuring stick" down at these points of the spectrum and test the scale. Assuming the value 10383.079 obtained from the first interval to be correct, it is seen that the wave-lengths at the end of the second interval (2800 Å) are about 0.003 Å too large, which is practically the same correction indicated by the combinations of terms B and S from Table I. When extended to the third interval it is evident that the wave-lengths near 2300 Å have been rather poorly determined.

A point of great interest is the relation between term combinations and behavior of the lines under pressure. During the progress of the spectrum analysis it was noticed that, so far as the pressure-shift of iron lines is known, the lines in any one multiplet belong to the same pressure group. The observations of increasing wave-length with pressure in the iron arc led Gale and Adams<sup>1</sup> to group the lines as follows: Group *a* shows small displacement, about 0.0013 Å per atmosphere at 4000 Å, 0.0026 Å at 5000 Å; group *b* changes 0.0021 Å per atmosphere at 4000 Å, 0.0043 Å at 5000 Å, and 0.0074 Å at 6000 Å; group *c*, about 0.0044 Å at 4000 Å, 0.0103 Å at 5000 Å; group *d*, 0.0084 Å at 4000 Å, 0.0142 Å at 5000 Å, and 0.025 Å at 6000 Å. The displacement is nearly proportional to the third power of the wave-length, and must be very large in the infra-red.

A comparison of pressure groups and term combinations shows that lines of group *a* belong to multiplets for which the combining spectral terms are greater than about 28,000 (cf. AF, AG, BF, BG, AE, CI), lines of group *b* belong to terms larger than about 19,000 (cf. BJ, BK, BL, CK, CM, CN, CP, DJ), while the *c* and *d* lines occur only in multiplets involving terms lower than Q (cf. FT, GT, HT, FU, HT', JT', KT', EX, VX, WX). The boundaries are not sharply drawn (DY consists of *b* lines but DQ of *c* lines), but it appears to be true in general that those lines which arise from electron transitions between outer orbits are most affected by pressure, and the amount of this pressure-shift is some function of the effective distances of the electron from its normal position. This

<sup>1</sup> *Astrophysical Journal*, **35**, 10, 1912; **37**, 391, 1913.

is exactly what might be expected since an increase of density in the radiating source would increase the probability that the higher-quantum electron orbits in the excited atom would be disturbed by collisions or other effects incident to the closer proximity of atoms.

This observation has important bearing on the accuracy in relative value of our system of wave-length standards. It was mentioned above that the so-called pole-effect in the iron arc led to the adoption of new specifications for this source, the principal features of which are increasing the length of the arc and taking light from a more restricted area in the center of the image. This pole-effect is probably related to pressure or some effect incident to vapor density gradients in the open-air arc, and it is doubtful, therefore, if a perfectly consistent system of standards can be derived even from the new arc. It has been stated that the iron lines to the red of 6000 Å are all stable (*a* and *b*) lines, and on this account the short arc has been retained as a source of standards in this spectral region.<sup>1</sup> This statement is now seen to be in error, the strong infra-red lines in multiplet DH no doubt belong to group *a*, but the red and infra-red lines in multiplets JU, KU, LU, and MU are just as certainly composed of *d* lines. If the pole-effect is related to the pressure-effect, and the latter increases as the cube of the wave-length, it may be expected that these red and infra-red *d* lines are subject to comparatively large displacements. No quantitative data on pole-effects for red lines exist, but these effects are readily computed from the term values in Table I since, as stated above, the spectral terms for *c* and *d* lines are based upon the wave-lengths obtained from the long arc, which was supposed to be free from this effect. In Table V are presented some calculations of pole-effect for the red and infra-red lines.

From what has been said it may be concluded that internal evidence from the structure of the spectrum shows that even the best values of wave-lengths now available from the iron arc in air do not form a perfect system of standards.

Aside from any question as to the absolute values of these standards there is the problem of establishing the relative values as a function of the smallest possible number of variables. The iron arc at atmospheric pressure gives values among which different

<sup>1</sup> *Trans. I.A.U.*, **1**, 36, 1922.

groups are affected differently by pressure and by pole-effects. These effects must be present in some measure even in the long arc, which reduces materially the differences due to the latter. The only way to eliminate these disturbing effects is to adopt as the source of secondary standards the iron arc *in vacuo*. Such a procedure will not only simplify the specifications for the source, but will lead to values which are simpler physical constants. It will, moreover, make possible a further refinement in the wave-length measurements because the sharpness of the lines is greatly improved in a vacuum source.

TABLE V  
CALCULATION OF "POLE-EFFECT"

Combination	$\mu$ Vac.	$\lambda$ Vac.	$\lambda$ Air Calc.	$\lambda$ Standard Obs.	$\lambda$ Obs.- $\lambda$ Calc.
J <sub>0</sub> -U <sub>1</sub> .....	14009.696	7092.351	7090.400	7090.416	0.016
J <sub>1</sub> -U <sub>1</sub> .....	14204.187	7040.178	7038.241	7038.260	.019
K <sub>3</sub> -U <sub>3</sub> .....	13426.753	7447.817	7445.770	7445.784	.014
K <sub>5</sub> -U <sub>5</sub> .....	13310.085	7513.100	7411.035	7511.053	.018
L <sub>5</sub> -U <sub>4</sub> .....	12595.517	7039.333	7037.153	7037.178	.025
L <sub>4</sub> -U <sub>3</sub> .....	12498.176	8001.168	7998.971	7998.986	.015
M <sub>5</sub> -U <sub>4</sub> .....	11998.734	8334.213	8331.926	8331.962	0.036

The spectrum of the iron arc is very prominent in the spectra of the sun, and because of its importance in this connection it is desirable to measure fundamental values characteristic of normal iron atoms. The best evidence now available indicates a very low pressure in the sun's atmosphere, and only vacuum-arc values of spectra may therefore be compared with the sun.

The logical source for standard wave-lengths is one which is free from pressure-effect, pole-effect, or any other incidental disturbances which must otherwise be controlled with elaborate specifications. It is believed that the iron arc *in vacuo* will furnish a more perfect and useful system of standards than the one now in use.

I am pleased to acknowledge indebtedness to Dr. F. M. Walters, Jr., who has placed at my disposal some unpublished multiplets and prepared the diagram of energy-levels and combinations.

WASHINGTON, D.C.  
BUREAU OF STANDARDS  
May 1, 1924

## REVIEWS

*A Treatise on the Analysis of Spectra.* By W. M. HICKS. Cambridge: The University Press, 1922. American agents: The Macmillan Company, New York. 8vo. Pp. 326. Figs. 25. \$11.00.

Poincaré has stressed the fact that there are an infinite number of ways of "explaining" any given set of phenomena, and that scientific "truth" is merely the simplest explanation as yet known. Viewed from this standpoint, the truth of any given theory is wholly a question of its ability (1) to correlate known facts and relations, (2) to predict additional facts and relations, thus stimulating research.

It has seemed to the reviewer that no field of scientific inquiry is more in need of theories than that of spectroscopy, especially that portion of spectroscopy dealing with the mathematical analysis of line spectra. The experimental data are of such complexity that it is almost impossible for anyone either to obtain or to retain a really comprehensive knowledge of the subject. For this reason any theory, reasonable or unreasonable, is of great value if, as noted in (1) above, it can provide a "framework" on which to hang spectroscopic facts. But, in addition, it has long been recognized that the mathematical relations of line spectra must provide the most powerful tool for the ultimate analysis of atomic structure. Hence the subject is of the greatest importance, and, as Professor Hicks remarks in the Introduction to his treatise, "It is much to be desired that a larger number of investigators should be drawn to this study." The chief handicap of these new workers thus far has been the lack of any adequate statement of the empirical facts. The volume under review is intended, according to the author, to provide such a compilation of facts.

The reviewer must confess, however, that he doubts if this purpose has been accomplished. A reading of the volume seems to indicate the sheer impossibility of presenting fairly the facts of spectroscopy without recourse to theory. Hicks has spent probably more time on spectral analysis than any other worker, and has presented a number of new relations, such as the determination of atomic weight by means of constants of spectral series (the "oun"), summation series, linkages, etc.

But these relations have not as yet been accepted by other spectroscopists, and it seems to the reviewer that there are grave doubts as to the validity of many of them. On the other hand, Hicks is at present admittedly a skeptic on the subject of quantum theory, mainly, it would appear, because it apparently contradicts certain of these new relations. It seems to the reviewer that relations such as summation series and linkages can easily be interpreted in terms of the complex levels now being found in all elements ("multiplets"), and that many of Hicks's new relations are destined to be "rediscovered" by future workers, while many others are probably spurious. Great stress, however, is laid by the author upon these topics, so that this work is not a systematic presentation of our present knowledge of line spectra (band spectra and X-ray spectra are not mentioned), but rather a compilation of the author's own work on the subject. It must be remembered, in this connection, that the book was actually written between 1919-1921, and since then many of the difficulties with which the author struggles have been definitely cleared up by means of Bohr's spatial atomic model.

Within the space of a review it is not possible to give any real synopsis of Hicks's own work. Such a summary is given on pages 51-58 of Fowler's *Report on Series in Line Spectra*. That report, and the Paschen-Götze volume on the same subject, were written mainly as a compilation of existing data. Hicks's treatise also contains such a compilation—in some respects more complete than the other two, in other respects less so. Hicks gives copious references, and also the Zeeman patterns of spectral lines, where these are known. He does not, however, give explicitly the spectral "terms" of which each line is composed. These may be obtained from the values of the series limits, given in a separate table, but it is very convenient to have these values listed with each line, as Fowler and Paschen-Götze have done.

Hicks apparently accepts the Bohr model of hydrogen and ionized helium (footnote, page 114), but his description of the hydrogen spectrum on pages 40-44 gives practically no hint of the remarkable agreement of theory and experiment. Aside from this case, the quantum theory seems to be introduced mainly for the purpose of criticism, and many of these criticisms are no longer valid, such as that on page 117, where the statement is made that no arc line can theoretically have a denominator less than unity, thus leaving unexplained the helium arc lines corresponding to excitation potentials greater than 13.5 volts. On quantum theory this fractional denominator means merely that for a given one-quantum orbit the effective charge is greater than unity,

as would be expected in any model of the helium atom. Similarly, beginning on page 56, his attempted explanations of the near infra-red alkali lines are now known to be incorrect. Hicks assumes the existence of only four possible sequence types, *s*, *p*, *d*, and *f*. But on quantum theory these correspond to azimuthal quantum numbers 1, 2, 3, and 4, respectively. It is quite possible to have higher values, and in fact the lines just mentioned are the most significant examples of new series involving the 5 and 6 valued sequences.

It is to be feared that the chief impression gained by the average reader of this work will be that little progress has as yet been made in unraveling the complexities of line spectra, and that the subject still offers but slight attraction to the new worker—an impression which is far from the truth at the present time. It must be admitted that many of the theoretical workers in quantum theory have had an insufficient knowledge of spectroscopic facts, and have therefore sometimes made unjustified claims as to the agreement of theory and experiment. But it is equally true that Hicks makes unjustified claims of the disagreement of theory and experiment, and the reviewer cannot conscientiously recommend this work to the theorist desirous of a correct statement of spectroscopic facts. What is still very much needed is a comprehensive statement of spectroscopic data, correlated in terms of modern theory. We have in abundance volumes giving the experimental verifications of quantum theory, and virtually ignoring all the unexplained experimental facts. The work under review represents the other extreme. What is needed is a synthesis of the two types.

RAYMOND T. BIRGE

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*Die Fernrohre und Entfernungsmesser.* By A. KÖNIG. Berlin: Julius Springer, 1923. 8vo. Pp. vii+207. Figs. 254. \$1.80, unbound; \$2.00, bound.

Dr. König's new book on telescopes and distance finders constitutes Volume V of a series of scientific monographs, entitled *Naturwissenschaftliche Monographien und Lehrbücher* and edited by the *Schriftleitung der "Naturwissenschaften."* Dr. König, through his connection with the optical firm of Carl Zeiss in Jena, is especially well qualified to be the author of a book on telescopes.

The book consists of three parts of unequal length. The first part, containing three chapters, deals with the theory of the telescope, and from an astronomical point of view the first two chapters constitute undoubtedly the most interesting portion of the whole book. The theory of the

lens and that of combinations of lenses are discussed briefly but in a clear and purely scientific manner. Of the greatest interest for an astronomer are the applications of the theory to the human eye, the theory of telescopic vision, and the discussion of distortion in optical images as produced by various forms of instruments. Mathematical details are set up in small print and may be readily omitted by those who are not interested in the mathematical developments.

Perhaps not quite enough space is devoted to the treatment of large refracting telescopes. Neither the Yerkes refractor nor the Lick refractor are mentioned in the Index, though they are referred to briefly on page 51. The big reflectors at the Mount Wilson Observatory are fully described, and photographs of some of the largest reflectors are reproduced on pages 80-83.

The last chapter of the first part deals with optical devices used in the aiming of guns and rifles. This chapter, as well as the entire third part of the book, which deals with distance finders, is of greater interest to an artillerist than to an astronomer.

The second part, consisting of only fifteen pages, is devoted to the description of various forms of micrometers.

The book makes a good appearance, the type is clear, and the illustrations are numerous and well chosen. A better grade of paper for the reproductions would have been an improvement, but would have increased the cost of the edition.

There are remarkably few mistakes or misprints. On page 85, line 31, the name "Snow" refers to Miss Helen Snow, who in 1903 donated to the Yerkes Observatory the Snow telescope, which was taken later to Mount Wilson.

OTTO STRUVE

*Ninth Report of the Section for the Observation of Variable Stars, 1915-1919.* Memoirs of the British Astronomical Association, Vol. XXV. C. L. BROOK, Director. Perth: Published for the Association, 1924. 8vo. Pp. v+554. Price to members, 5s.; non-members, 7s. 6d.

This valuable, though belated, report contains 27,820 observations of 36 variable stars of long period, giving an average of 146 per star per year. Only three of the stars are south of the equator, Mira, R Hydrae, and S Virginis; and only five were discovered as late as 1900. Of the thirty-two observers fourteen are located outside of England.

The Introduction was written by Professor H. H. Turner. The observations are tabulated for each variable in chronological order,

prefaced by notes on deviations from the magnitudes of the comparison stars given in *Harvard Annals*, 37. The tables give the date, instrument, observer, individual comparisons, deduced magnitude, and remarks. Colonel Markwick has undertaken the drawing of the curves and discussion of the observations. Table B gives an index of the number of observations of each variable by each observer.

J. A. PARKHURST

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*Monthly Report of the American Association of Variable Star Observers, 1923.* HOWARD O. EATON, Recording Secretary. Reprinted from *Popular Astronomy*, Vol. XXXI. 8vo. Pp. 129.

The title is inexact, as this is an annual compilation of the monthly reports, covering the period from October 20, 1922, to October 20, 1923. On page 114 is given a summary of the annual reports which indicates that for the year ending September 20, 1923, there were 17,745 comparisons of 441 stars by 160 observers; truly a remarkable record. The average number of observations per star was 40. The work, as taken from the pages of *Popular Astronomy*, is printed for each month separately. The stars are arranged in order of right ascension and the tables give the designation and position of the star, the Julian date to one decimal place, the estimated magnitude, and the initial of the observer. This condensed form makes it possible to report such a great number of observations in a small space.

A comparison between the British and American reports will be of interest. Of the latter, five times as many observers watched twelve times as many stars, giving them an average of a quarter as many observations per year. The American observers made an average of 261, the British, 174, comparisons per year. The large American observing list, 441 stars, contains most of the new variables. The advantage of the British report lies in giving the detailed comparisons, of the American, that it contains the greater volume of work. Each adheres closely to the Harvard magnitude scale. It is expected that a discussion of the American work will appear in the *Harvard Annals*.

It is difficult to overestimate the importance of such reports as these. The study of stellar variation ranks in importance with stellar spectroscopy; and the great amount of data now being accumulated will doubtless yield results of the highest value.

J. A. PARKHURST